Cosmological Implications of Spontaneously Breaking Lorentz Symmetry

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Einstein-Aether Theories

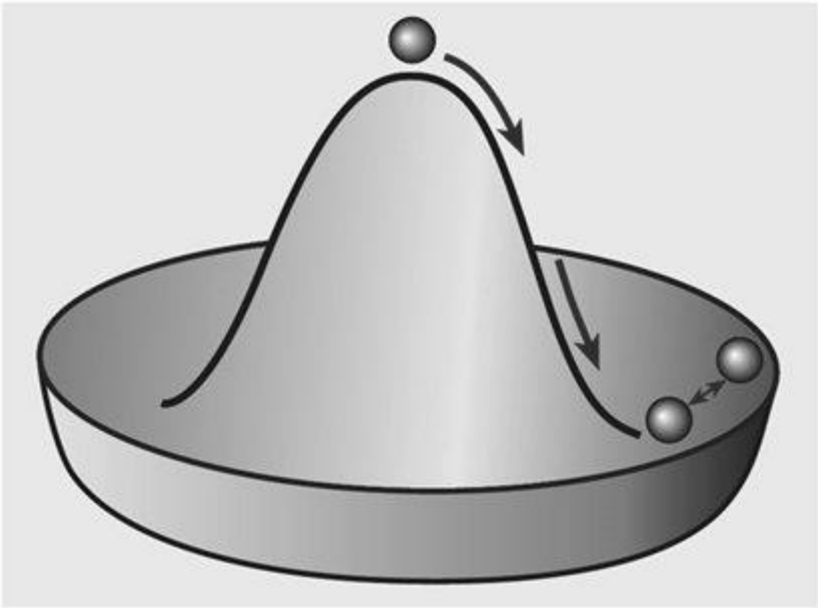
$$S = \frac{1}{2\,\kappa} \int \sqrt{-g}\left(-R - K^{ab}_{mn} \nabla_a u^m \nabla_b u^n - \lambda(g_{ab} u^a u^b - 1)\right)$$

$$K^{ab}_{mn} = c_1 g^{ab} g_{mn} + c_2 \delta^a_m \delta^b_n + c_3 \delta^a_n \delta^b_m + c_4 u^a u^b g_{mn}$$

- presence of a vector field
- is *not* Lorentz invariant
- the aether does not couple to curvature

Spontaneous Symmetry Breaking

- potential rolls to its *vev*
- Bumblebee vector acquires a space-time orientation
- Universe is endowed with a vector pointing in a specific direction
- this breaks Lorentz symmetry



The Bumblebee Model

$$S = \int \sqrt{-g}\left[\frac{1}{2\,\kappa}\left(R + \xi\, B^{\mu}\, B^{\nu}\, R_{\mu\nu}\right) - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} - V\left(B^{\mu} B_{\mu} \pm b^{2}\right)\right] d^{4}x + S_{M}$$

[1] Kostelecký (2004)

- vector field couples to curvature
- theory incorporates an arbitrary potential, V
- spontaneous symmetry breaking mechanism

Field and Bumblebee Equations

$$G_{\mu\nu} = \kappa\left(-B_{\mu\alpha}\, B^{\alpha}{}_{\nu} - \frac{1}{4}B_{\alpha\beta}B^{\alpha\beta}g_{\mu\nu} - V\, g_{\mu\nu} + 2\, V' B_{\mu}B_{\nu}\right)$$
$$+ \xi\left(\frac{1}{2}B^{\alpha}B^{\beta}R_{\alpha\beta}\, g_{\mu\nu} - B_{\mu}B^{\alpha}R_{\alpha\nu} - B_{\nu}B^{\alpha}R_{\alpha\mu} + \frac{1}{2}\nabla_{\alpha}\nabla_{\mu}B^{\alpha}B_{\nu} + \frac{1}{2}\nabla_{\alpha}\nabla_{\nu}B^{\alpha}B_{\mu}\right.$$
$$\left. - \frac{1}{2}\nabla_{\alpha}\nabla_{\beta}B^{\alpha}B^{\beta}g_{\mu\nu} - \frac{1}{2}\Box B_{\mu}B_{\nu}\right) - T_{\mu\nu}$$

- mixed terms in every component
- strutcture is richer than an average scalar field theory

$$\nabla_{\mu}B^{\mu\nu} = 2\left(V'B^{\nu} - \frac{\xi}{2\kappa}B_{\mu}R^{\mu\nu}\right)$$

- derivative of the potential with respect to its argument

Cosmology

Cosmological Approach
Dynamical Analysis

Selection of the Metric

- Bumblebee vector – non-trivial only on the temporal component

$$B_\mu = (B_t(t),\ 0,\ 0,\ 0)$$

- Homogeneity, isotropy and spherical symmetry - FLRW metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2\left[\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 + r^2\sin^2(\theta)\,\mathrm{d}\phi^2\right]$$

Modified Field Equations

- Modified Friedmann Equation: $$\frac{\kappa}{3}(\rho + V) + H^2(\xi B^2 - 1) + \xi B\dot{B}H = 0$$
- Temporal Bumblebee Equation: $$2\kappa V' B^2 = 3\xi \frac{\ddot{a}}{a} B^2$$
- Modified Raychaudhuri Equation:

$$\left(2\dot{H} + 3H^2\right)(\xi B^2 - 1) + \xi\left[(\dot{B})^2 + 4B\dot{B}H + B\ddot{B}\right] + \kappa V = 0$$

- Equation of Modified Conservation of Energy:

$$\dot{\rho} = -3H\rho + 3\frac{\xi}{\kappa}\frac{\ddot{a}}{a}B^2 - 2(B^2 H + B\dot{B})V'$$

General de Sitter Solution

$$a(t) = a_0 \, e^{H_0 (t - t_0)}$$

$$\frac{\kappa}{V}(\rho + V) + H^2(\xi B^2 - 1) + \xi B \dot{B} H = 0$$

$$V' B^2 = \frac{3}{2}\frac{\xi}{\kappa}\frac{\ddot{a}}{a} B^2$$

$$(2\dot{H} + 3H^2)\,(\xi B^2 - 1) + \xi\left[\dot{B}^2 + 4B\dot{B}H + B\ddot{B}\right] + \kappa V = 0$$

$$\dot{\rho} + 3H\rho = 3\frac{\xi}{\kappa}\frac{\dddot{a}}{a} B^2 - 2\left(HB^2 + B\dot{B}\right)V'$$

General de Sitter Solution

$$\frac{\kappa}{V}(\rho + V) + {H_0}^2(\xi B^2 - 1) + \xi B\dot{B}H_0 = 0$$

$$\boxed{V' B^2 = \frac{3}{2}\frac{\xi}{\kappa}{H_0}^2 B^2} \longrightarrow \underline{B \text{ constant}}$$

$$(2\cancel{\ddot{H}} + 3{H_0}^2)\,(\xi B^2 - 1) + \xi\left[\dot{B}^2 + 4B\dot{B}H_0 + B\ddot{B}\right] + \kappa V = 0$$

$$\dot{\rho} + 3H_0\rho = 3\frac{\xi}{\kappa}{H_0}^3 B^2 - 2\left(H_0 B^2 + B\dot{B}\right)V'$$

General de Sitter Solution

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General de Sitter Solution

$$\frac{\kappa}{V}(\cancel{\rho} + V) + {H_0}^2(\xi B^2 - 1) + \xi B\cancel{\dot{B}}H_0 = 0$$

vanishing density

$$V'B^2 = \frac{3}{2}\frac{\xi}{\kappa}{H_0}^2B^2$$

→ B constant

$$(2\cancel{\dot{H}} + 3{H_0}^2)\,(\xi B^2 - 1) + \xi\left[\cancel{\dot{B}^2} + 4B\cancel{\dot{B}}H_0 + B\cancel{\ddot{B}}\right] + \kappa V = 0$$

$$\cancel{\dot{\rho}} + 3\cancel{H_0}\rho = 3\frac{\xi}{\kappa}{H_0}^3B^2 - 2\left(H_0B^2 + \cancel{B}\dot{B}\right)V'$$

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$$\dot{\rho} + 3H_0\rho = 3\frac{\xi}{\kappa}{H_0}^3B^2 - 2\left(H_0B^2 + B\dot{B}\right)V'$$

same equation

General de Sitter Solution

$$H_0{}^2 = \frac{\kappa V}{3(1-\xi B^2)}$$

$$\frac{\kappa}{V}(\rho + V) + H_0{}^2(\xi B^2 - 1) + \xi B\dot{B}H_0 = 0$$

B constant

vanishing density

$$V'B^2 = \frac{3}{2}\frac{\xi}{\kappa}H_0{}^2B^2$$

$$(2\dot{H} + 3H_0{}^2)\,(\xi B^2 - 1) + \xi\left[\dot{B}^2 + 4B\dot{B}H_0 + B\ddot{B}\right] + \kappa V = 0$$

$$\dot{\rho} + 3H_0\rho = 3\frac{\xi}{\kappa}H_0{}^3B^2 - 2\left(H_0B^2 + B\dot{B}\right)V'$$

same equation

General de Sitter Solution

$$H_0{}^2 = \frac{\kappa V}{3(1 - \xi B^2)}$$

$$V' B^2 = \frac{3}{2}\frac{\xi}{\kappa} H_0{}^2 B^2$$

$$V = A(B^2 \pm b^2)^{\,n} \quad \rightarrow \quad V' = \frac{nV}{B^2 \pm b^2} \quad \rightarrow \quad \frac{nV}{B^2 \pm b^2} = \frac{3}{2}\frac{\xi}{\kappa} H_0{}^2$$

$$\frac{2n(1 - \xi B^2)}{B^2 \pm b^2} = \xi \quad \rightarrow \quad 2n(1 - \xi B^2) = \xi(B^2 \pm b^2) \quad \rightarrow$$

$$\xi B^2 = \frac{2n \mp \xi b^2}{1 + 2n}$$

$$\frac{\kappa V}{3H_0{}^2} = 1 - \xi B^2 = \frac{1 \pm \xi b^2}{1 + 2n}$$

General de Sitter Solution

$$S = \int \sqrt{-g}\left[\frac{1}{2\,\kappa}\left(R + \xi\, B^{\mu}\, B^{\nu}\, R_{\mu\nu}\right) - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} - V\left(B^{\mu} B_{\mu} \pm b^{2}\right)\right] d^{4}x + S_{M}$$

- on-shell action:

$B = 0$

$$S = \int \sqrt{-g}\left[\frac{1}{2\,\kappa} R - V(\pm b^{2})\right] d^{4}x$$

$$= \int \sqrt{-g}\left[\frac{1}{2\,\kappa} R - \frac{3{H_0}^{2}}{\kappa}\right] d^{4}x$$

$B \neq 0$

$$S = \int \sqrt{-g}\left[\frac{1}{2\,\kappa} R + \frac{3{H_0}^{2}}{\kappa}\left(\frac{2n \mp \xi b^{2}}{1+2n} + \frac{2 \pm 2\,\xi b^{2}}{1+2n}\right)\right] d^{4}x$$

$$= \int \sqrt{-g}\left[\frac{1}{2\,\kappa} R + \frac{3{H_0}^{2}}{2\,\kappa}\left(1 + \frac{1 \pm \xi b^{2}}{1+2n}\right)\right] d^{4}x$$

Dynamical Analysis

Definition of adimensional variables:

$$x_1 = \frac{\kappa V}{3H^2} \qquad x_2 = \xi B^2 \qquad x_3 = \frac{\xi B \dot{B}}{H} \qquad \Omega_M = \frac{\kappa \rho}{3H^2} \qquad q = -\frac{a\,\ddot{a}}{\dot{a}^2}$$

Rewritten Equations:

$$x_1' = 2(1 + \alpha x_3 - 2\alpha x_1)x_1$$

$$x_2' = 2x_3$$

$$x_3' = (1 + 4\alpha x_1)(1 - x_2) - 3x_1 - x_3(3 + 2\alpha x_1)$$

$$1 = x_1 + x_2 + x_3 + \Omega_M$$

Fixed Points

Four fixed points:

$$(x_1, x_2, x_3, \Omega_M, q) = (1,0,0,0,-1)$$

$$(x_1, x_2, x_3, \Omega_M, q) = (0,0,0,1,\frac{1}{2})$$

$$(x_1, x_2, x_3, \Omega_M, q) = (0,1,0,0,0)$$

$$(x_1, x_2, x_3, \Omega_M, q) = (\frac{1+\xi b^2}{1+2n}, \frac{2n-\xi b^2}{1+2n}, 0,0,-1)$$

- reobtained the previously derived solutions

Stability

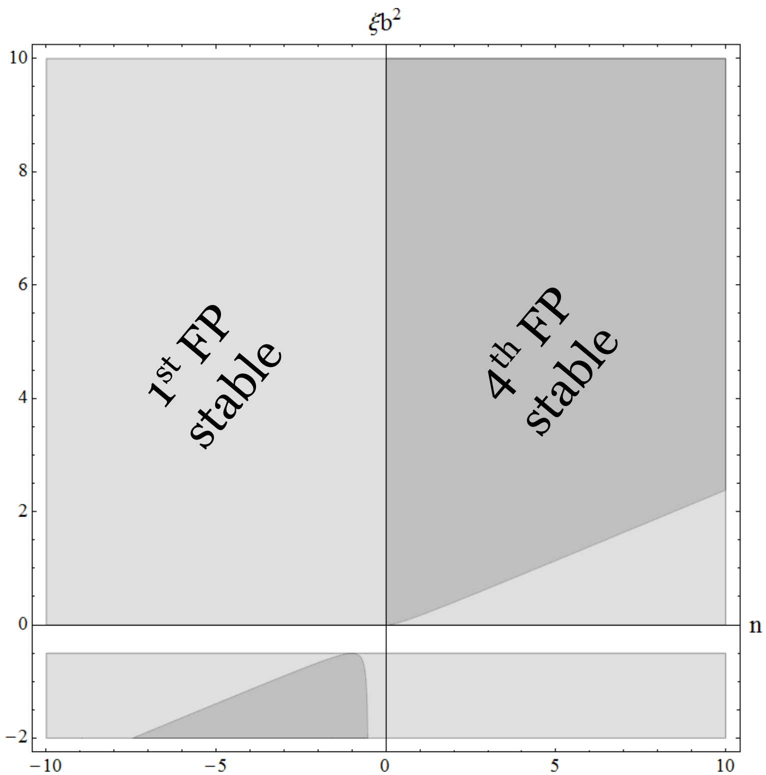
$$(1,0,0,0,-1)$$

$$(0,0,0,1,\frac{1}{2})$$

$$(0,1,0,0,0)$$

$$(\frac{1+\xi b^2}{1+2n},\frac{2n-\xi b^2}{1+2n},0,0,-1)$$

- one attractor and one repulsor
- two repulsors
- two atractors

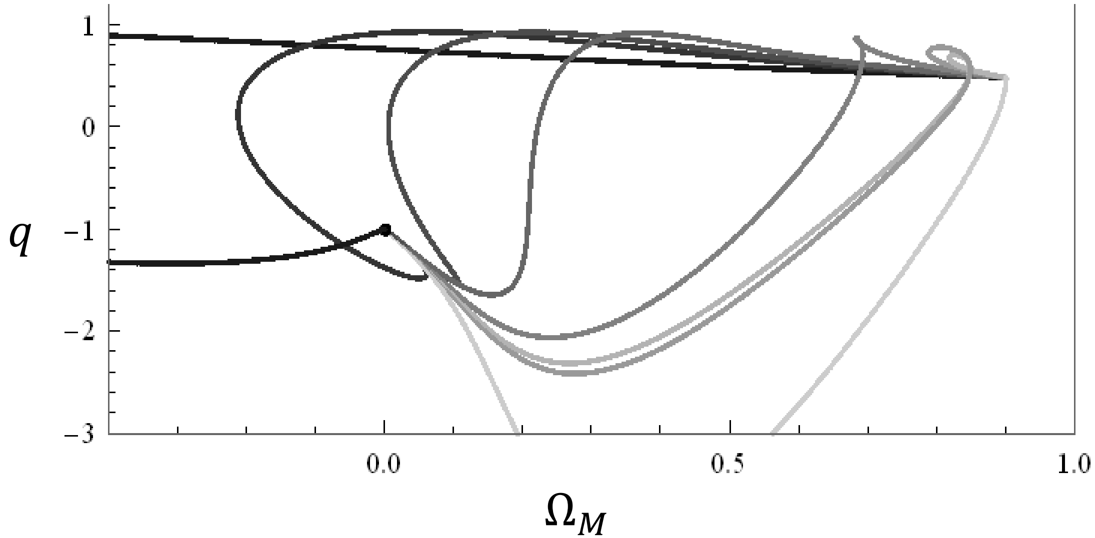


Plot of parameter values for: (light) 1st fixed point stable; (dark) 4th fixed point stable

Results and Conclusions

Evolution of the Universe
Parameter Determination
Conclusions and Outlook
References

n = 2 (convergence)



Projection of the tridimensional phase-space onto the $\Omega_M - q$ plane.
The dot denotes the system's attractor.

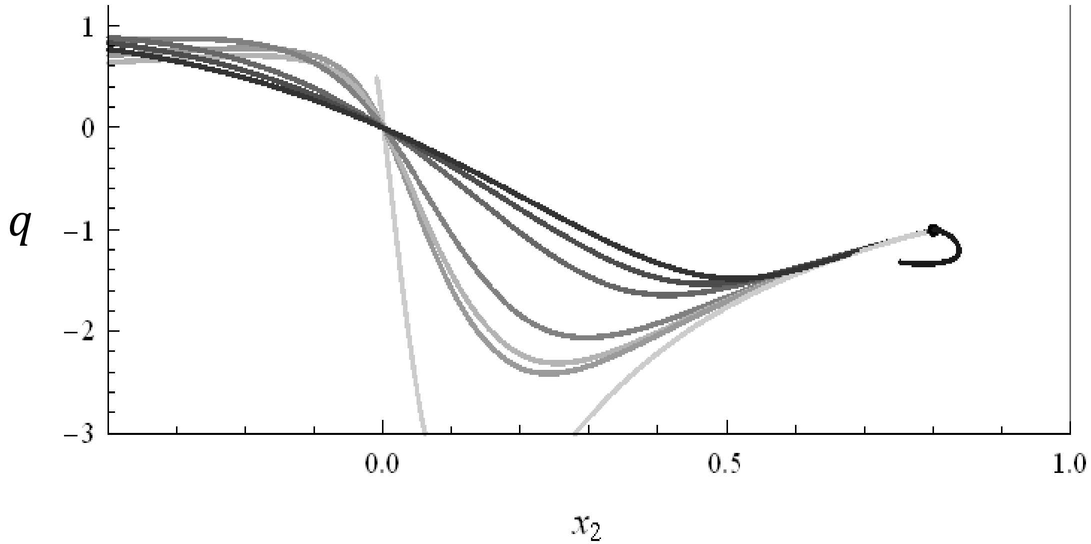
initial values

$x_4 = 0.9, \quad x_5 = 0.49$

$\xi b^2 = 10^{-12}$

- $x_2 = -6$
- $x_2 = -3.5$
- $x_2 = -3$
- $x_2 = -2.5$
- $x_2 = -1.5$
- $x_2 = -1$
- $x_2 = -0.8$
- $x_2 = -0.01$

$n = 2$ (convergence)



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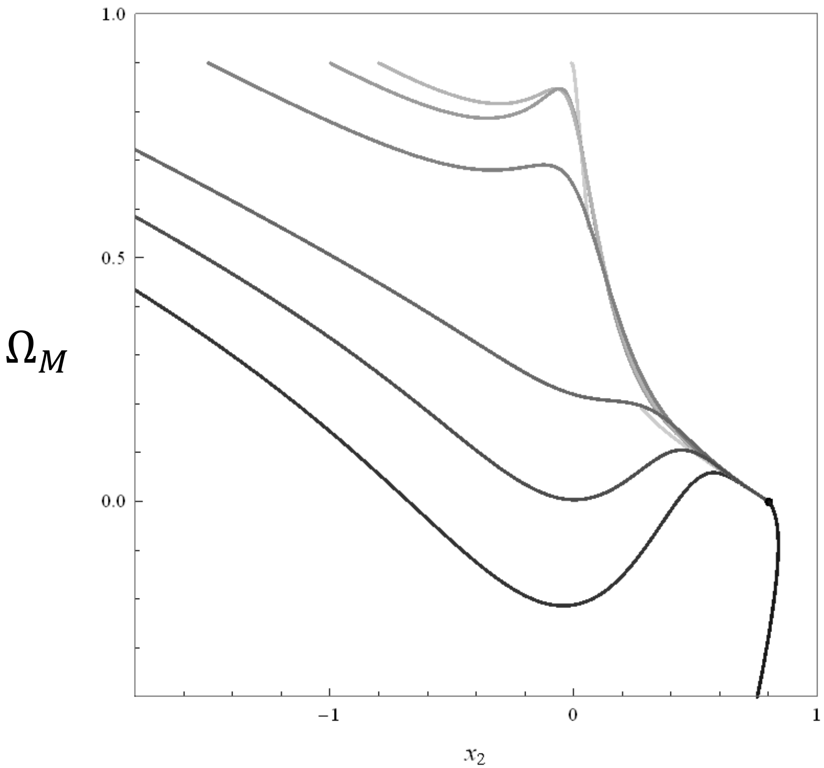

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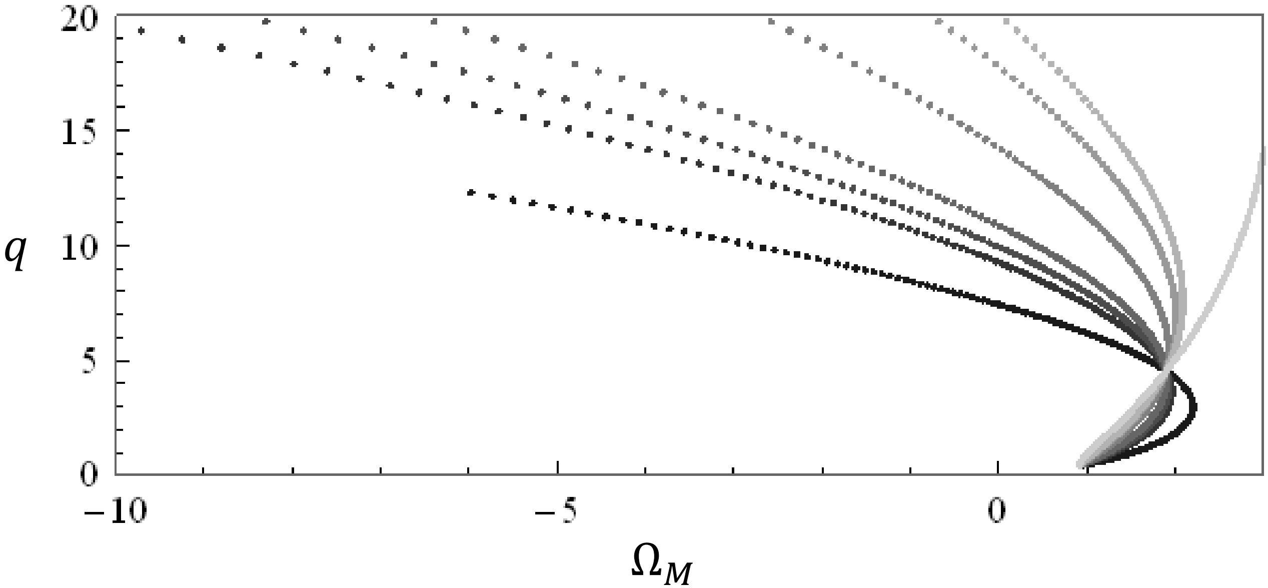
n = 1 (divergence)



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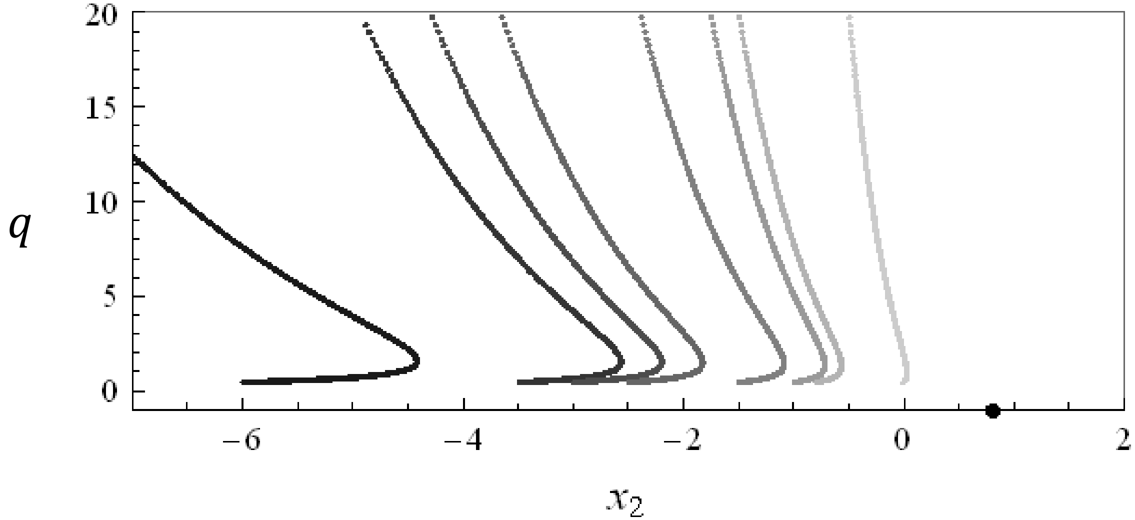

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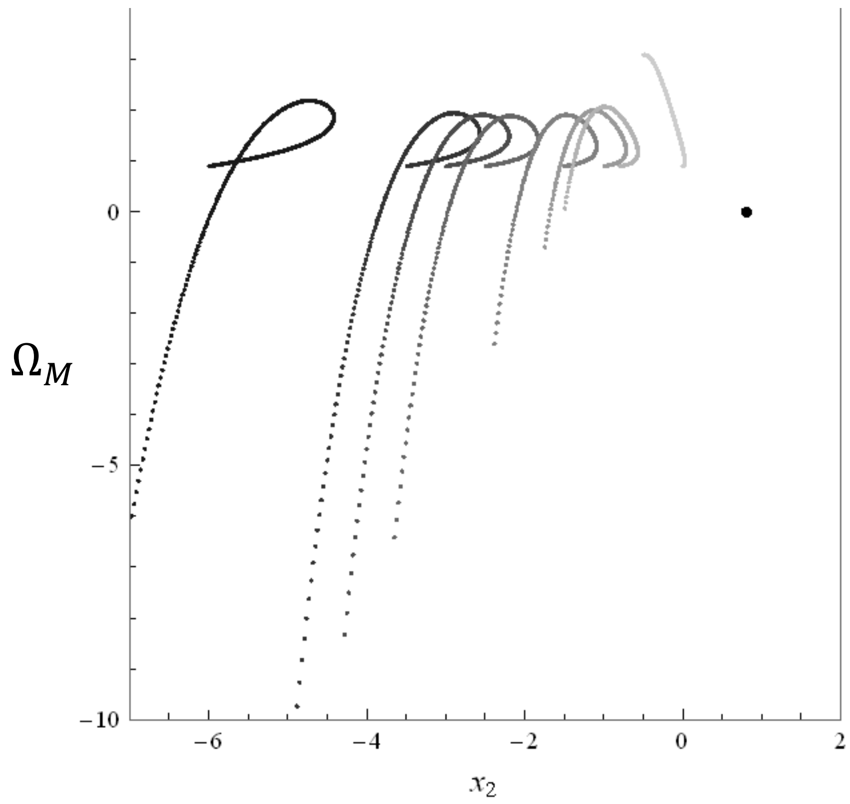

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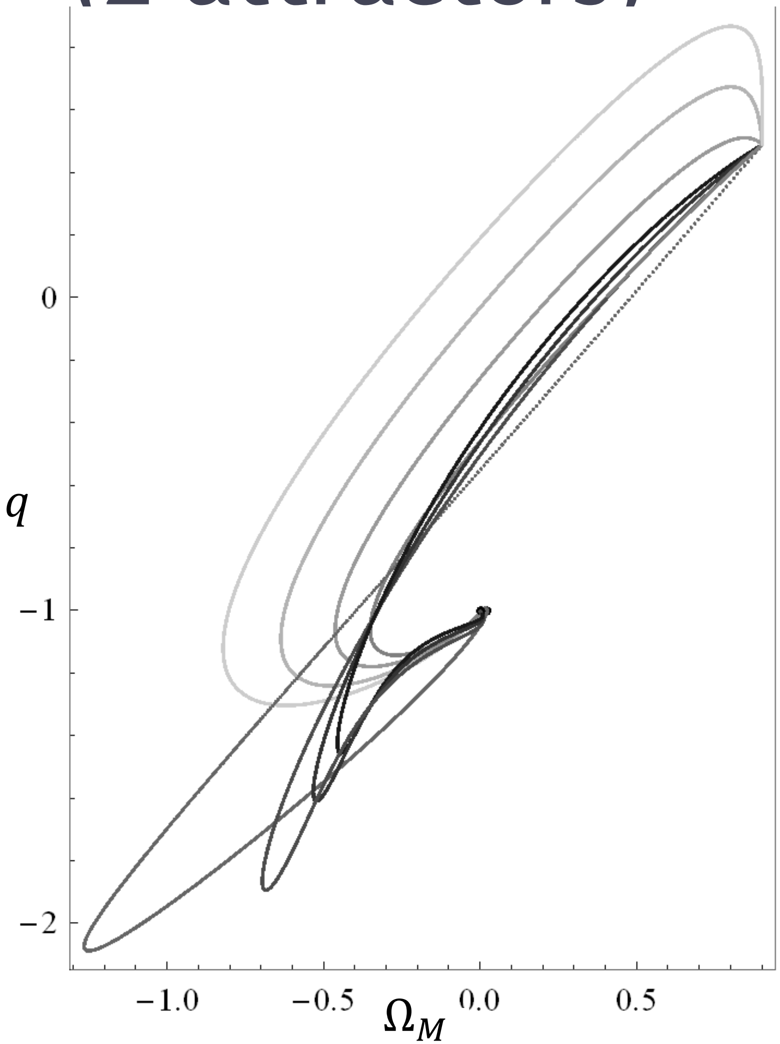
Even Oscillating Potential (2 attractors)



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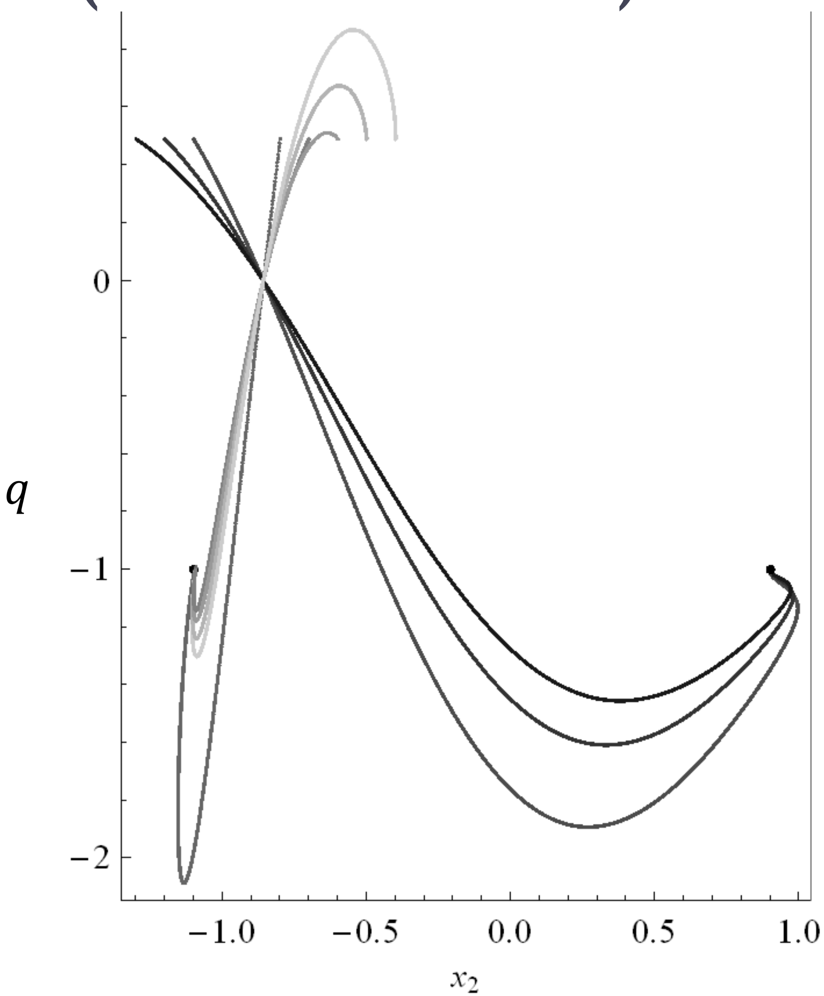
- $x_2 = -0.8$
- $x_2 = -0.7$
- $x_2 = -0.6$
- $x_2 = -0.5$
- $x_2 = -0.4$

Even Oscillating Potential (2 attractors)



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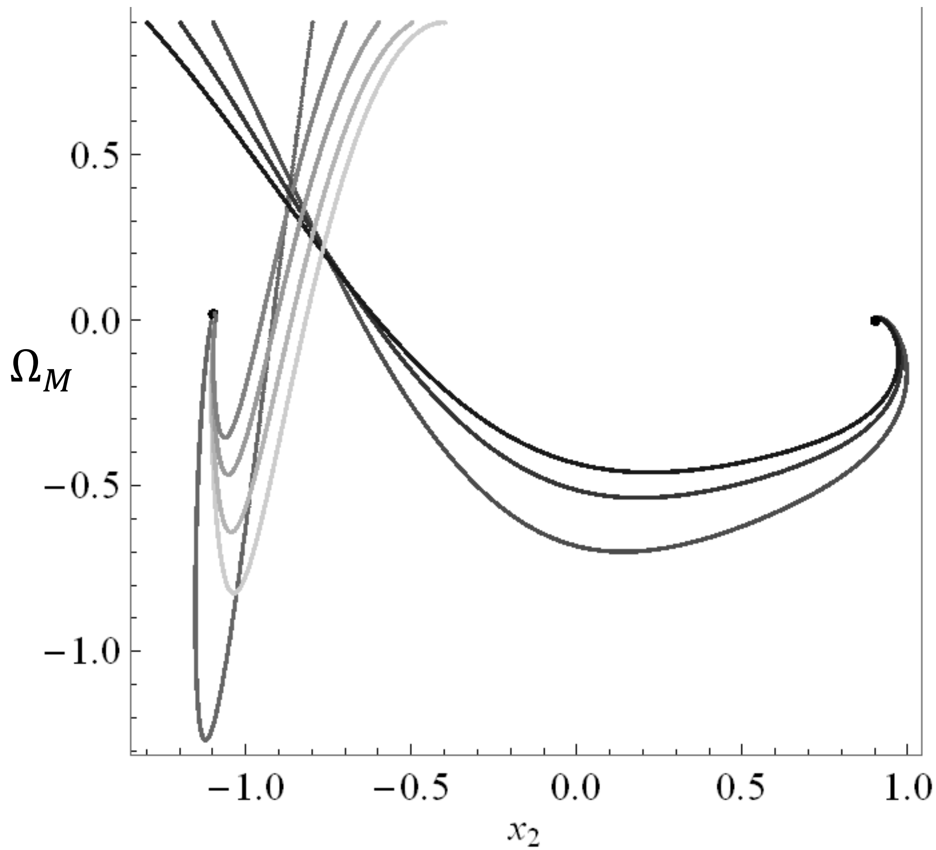

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- $x_2 = -0.6$
- $x_2 = -0.5$
- $x_2 = -0.4$

Conclusions

- correctly predicts the currently expected evolution of the Universe
- replicates deceleration parameter transition from positive to negative: compatible with the (over-)accelerated expansion of the Universe
- provides a possible candidate to dark energy
- theory is flexible: freedom to choose the potential, freedom to choose the directions of symmetry breaking
- high complexity level, possibility to generalize

References

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