Extended theories of gravity and the late-time cosmic acceleration

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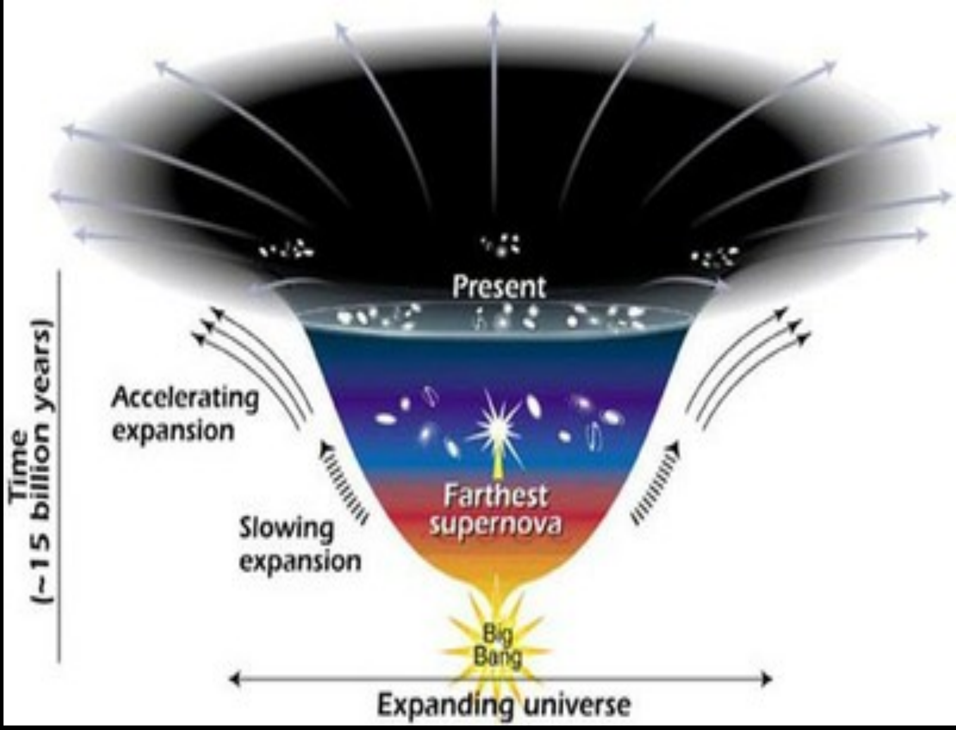


- Cosmology is thriving in a golden age, where a central theme is the perplexing fact that the Universe is undergoing an accelerating expansion.

- This represents a new imbalance in the governing gravitational equations.

  Cause? Remains an open and tantalizing question

- Historically, physics has addressed such imbalances by either identifying sources that were previously unaccounted for, or by altering the governing equations.

- The standard model of cosmology has favored the first route to addressing the imbalance by a missing stress-energy component.

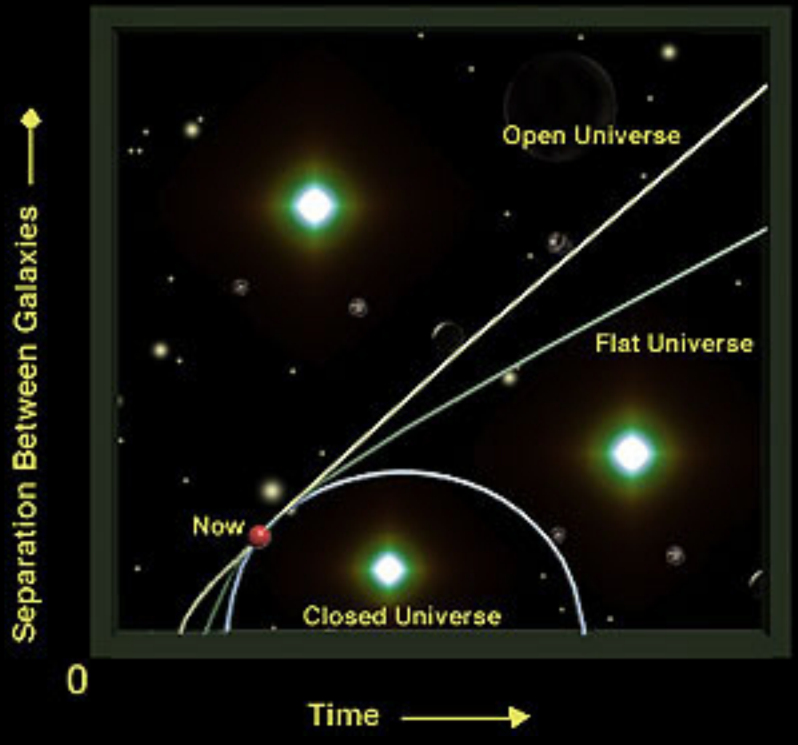
- One may also explore the alternative viewpoint, namely, through a modified gravity approach.

1. Cosmology in a nutshell

Assume the Universe to be spatially homogeneous and isotropic, governed by the FLRW metric:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

k **= -1,0,+1 (open, flat, or closed Universe)**



Matter content described by a perfect fluid:

$$T^{\mu\nu} = (\rho + p)U^\mu U^\nu + p g_{\mu\nu}$$

ρ = energy density,

p = uniform pressure

Einstein field equation:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Ordinary matter obeys Energy Conditions:

$$1.\ \mathrm{NEC}: \rho + p \geq 0$$

$$2.\ \mathrm{WEC}: \rho \geq 0, \rho + p \geq 0$$

$$3.\ \mathrm{SEC}: \rho + p \geq 0, \rho + 3p \geq 0$$

$$4.\ \mathrm{DEC}: \rho \geq |p|$$

Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2},$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

Accelerated expansion: Violates the SEC!!

$$\ddot{a} > 0$$

$$\rho + 3p < 0$$

Introduce cosmological constant → vacuum energy:

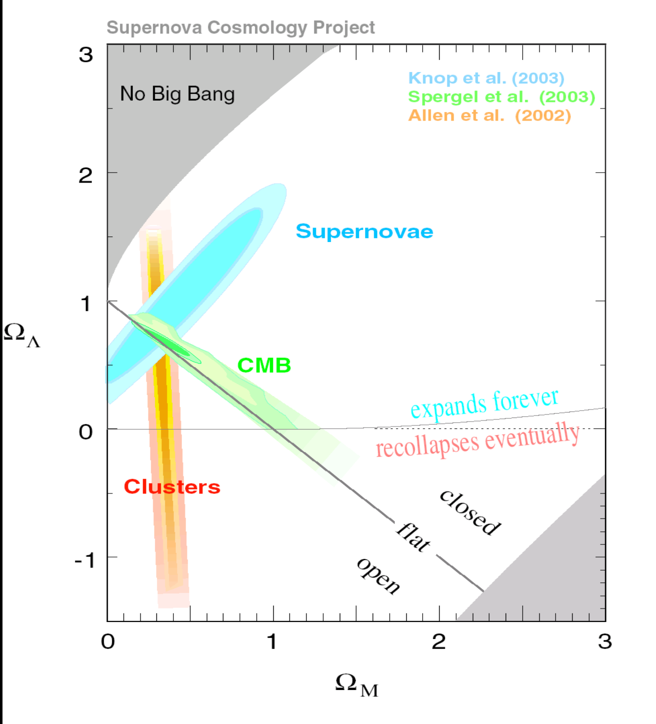
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$T_{\mu\nu}^{(vac)} = -\frac{\Lambda}{8\pi G} g_{\mu\nu}$$

ΛCDM model

2. ΛCDM fits data well, but we cannot explain it

- Simplest model
- Compatible with all data so far
- No other model is a better fit
- However, theory cannot explain it!



$$\rho_\Lambda\big|_{\rm obs} = \frac{\Lambda}{8\pi G} \sim H_0^2 M_p^2 \sim (10^{-33}\ \mathrm{eV})^2 (10^{28}\ \mathrm{eV})^2 \sim (10^{-3}\ \mathrm{eV})^4$$

$$\rho_\Lambda\big|_{\rm theory} = \text{vacuum energy} \sim (10^{27}\ \mathrm{eV})^4 >> \rho_\Lambda\big|_{\rm obs}$$

- Why so small? (note a difference of 120-orders-of-magnitude. EMBARRASSING!!!)
- Why so fine-tuned?

Alternatives to ΛCDM?

Dynamical Dark Energy in General Relativity (GR) "quintessence", etc…

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + 8\pi G T_{\mu\nu}^{\text{dark}}$$

$$T_{\mu\nu}^{\text{dark}} = \text{ time-varying DE field}$$

$$\omega = \frac{p_{\text{DE}}}{\rho_{\text{DE}}} < -\frac{1}{3}$$

Dark Gravity – Modify GR on large scales

$$G_{\mu\nu} + G_{\mu\nu}^{\text{dark}} = 8\pi G T_{\mu\nu}$$

$$G_{\mu\nu}^{\text{dark}} = \text{new scalar}$$

$$\text{to modify expansion}$$

General Relativity (GR): Hilbert-Einstein action

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa}R + L_m(g^{\mu\nu},\psi)\right]$$

- GR is a classical theory, therefore no reference to an action is required;
- However, the Lagrangian formulation is elegant, and has merits:
  1. Quantum level: the action acquires a physical meaning, and a more fundamental theory of gravity will provide an effective gravitational action at a suitable limit;
  2. Easier to compare alternative gravitational theories through their actions rather than by their field equations (the latter are more complicated);
  3. In many cases one has a better grasp of the physics as described through the action, i.e., couplings, kinetic and dynamical terms, etc

General Relativity (GR): Hilbert-Einstein action

Consider a 4-dimensional manifold with a connection, $\Gamma^{\alpha}{}_{\mu\nu}$ and a symmetric metric $g_{\mu\nu}$.

- The affine connection is the Levi-Civita connection. This requires that:
  1. The metric to be covariantly conserved;
  2. The connection to be symmetric.
- No fields other than the metric mediate the gravitational interaction;
- Field equations should be 2nd order partial differential equations;
- Field equations should be covariant (or the action be diffeomorphism invariant).

Higher order gravity:

Higher order actions may include various curvature invariants, such as:

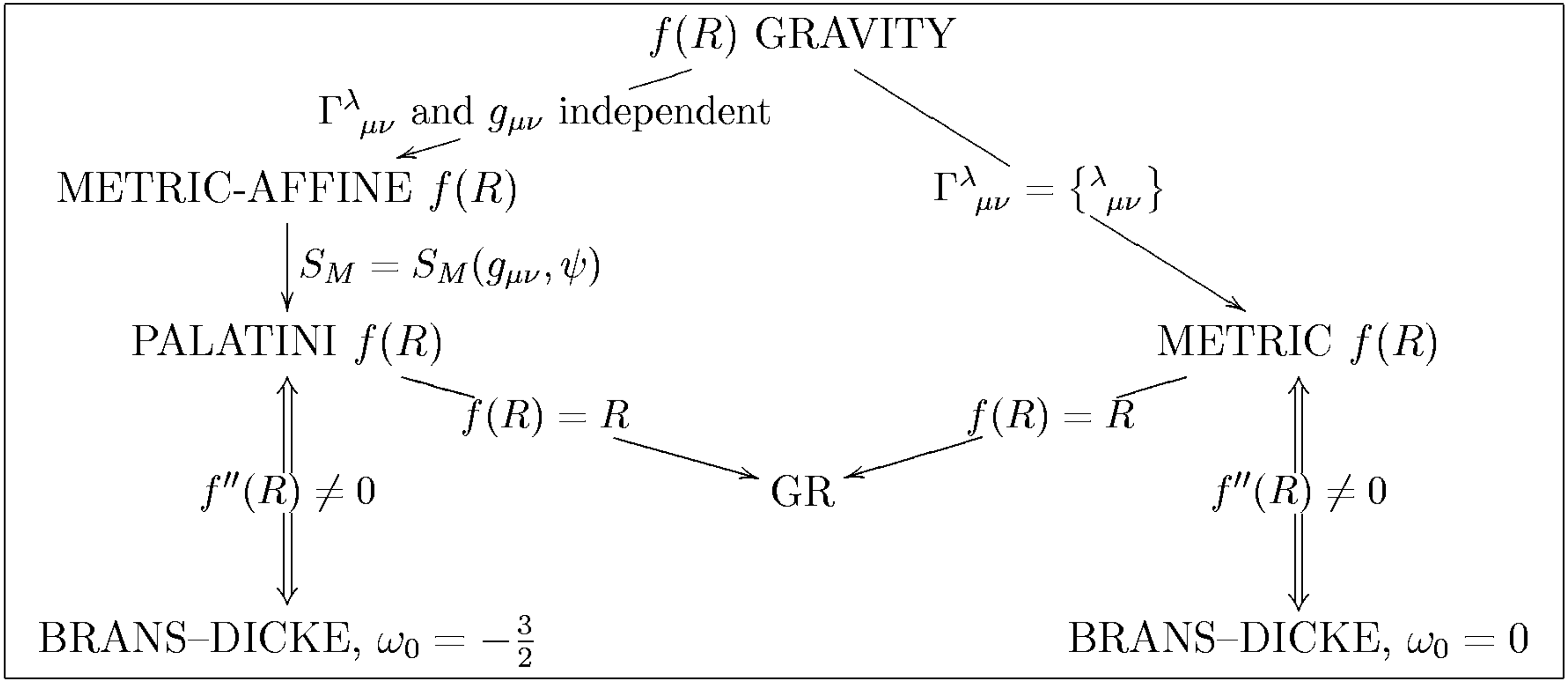
$$R^2, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}, etc.$$

Consider f(R) gravity, for simplicity:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa}f(R) + L_m(g^{\mu\nu},\psi)\right]$$

Appealing feature: combines mathematical simplicity and a fair amount of generality!

3. f(R) gravity: The equivalence of theories



3. f(R) gravity

Action:

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa} f(R) + L_m(g^{\mu\nu}, \psi)\right]$$

Gravitational field equations (vary action with $g^{\mu\nu}$):

$$F R_{\mu\nu} - \frac{1}{2} f g_{\mu\nu} - \nabla_\mu \nabla_\nu F + g_{\mu\nu} \nabla_\alpha \nabla^\alpha F = \kappa T^{(m)}_{\mu\nu}, \quad F = \frac{df}{dR}$$

Effective Einstein equation: $G_{\mu\nu} = \kappa T^{eff}_{\mu\nu}$ **with** $T^{eff}_{\mu\nu} = T^{(c)}_{\mu\nu} + \overline{T}^{(m)}_{\mu\nu}$

$$\overline{T}^{(m)}_{\mu\nu} = T^{(m)}_{\mu\nu} / F, \quad \text{and} \quad T^{(c)}_{\mu\nu} = \frac{1}{\kappa F}\left[\nabla_\mu \nabla_\nu F - \frac{1}{4} g_{\mu\nu}\left(RF + \nabla_\mu \nabla^\mu F + \kappa T\right)\right]$$

Conservation law:

$$\nabla^\mu T^{(c)}_{\mu\nu} = \frac{1}{F^2} T^{(m)}_{\mu\nu} \nabla^\mu F$$

Ricci scalar is a dynamical degree of freedom:

$$\boxed{FR - 2f + 3\nabla^{\mu}\nabla_{\mu}F = \kappa T}$$

Introduces a new light scalar degree of freedom.

Consider: $\boxed{f(R) = R - \frac{\mu^4}{R}, \quad \mu \sim H_0}$ at low curvatures, 1/*R* dominates

This produces late-time self-acceleration

- But the light scalar strongly violates solar system constraints.
- All f(R) models have this problem
- Way out: ‘chameleon’ mechanism, i.e. the scalar becomes massive in the solar system

  - very contrived!!

3.1 R-matter couplings in f(R) gravity

Action:

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa} f_1(R) + \left[1 + \lambda f_2(R)\right] L_m \right\}$$

Gravitational field equations:

$$F_1 R_{\mu\nu} - \frac{1}{2} f_1 g_{\mu\nu} - \nabla_\mu \nabla_\nu F_1 + g_{\mu\nu} \nabla_\alpha \nabla^\alpha F_1 = -2\lambda F_2 L_m R_{\mu\nu}$$
$$+ 2\lambda(\nabla_\mu \nabla_\nu - g_{\mu\nu} \nabla_\alpha \nabla^\alpha) L_m F_2 + (1 + \lambda F_2) T_{\mu\nu}^{(m)}$$

Using the generalised Bianchi identities, one has a corrected conservation equation:

$$\nabla^\mu T_{\mu\nu}^{(m)} = \frac{\lambda F_2}{1 + \lambda f_2} \left[g_{\mu\nu} L_m - T_{\mu\nu}^{(m)} \right] \nabla^\mu R$$

(Bertolami, Boehmer, Harko, FL, PRD 2007)

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Gravitational field equations:

$$F_1R_{\mu\nu}-\frac{1}{2}f_1g_{\mu\nu}-\nabla_\mu\nabla_\nu F_1+g_{\mu\nu}\nabla_\alpha\nabla^\alpha F_1=-2\lambda F_2L_mR_{\mu\nu}$$
$$+2\lambda(\nabla_\mu\nabla_\nu-g_{\mu\nu}\nabla_\alpha\nabla^\alpha)L_mF_2+(1+\lambda F_2)T^{(m)}_{\mu\nu}$$

(Perfect fluid) Non-geodesic motion: with an "extra force":

$$\frac{dU^\mu}{ds}+\Gamma^\mu_{\alpha\beta}U^\alpha U^\beta=f^\mu$$

$$f^\mu=\frac{1}{(\rho+p)}\left[\frac{\lambda F_2}{1+\lambda f_2}(L_m-p)\nabla_\nu R+\nabla_\nu p\right]h^{\mu\nu}$$

$$h_{\mu\nu}=g_{\mu\nu}+U_\mu U_\nu$$

Intriguing result: Field eqs. depend on matter Lagrangian.

Different choices:

$$L_m=p,\; L_m=-\rho$$

(Bertolami, FL, Paramos, PRD 2008)

3.2 Late-time cosmic acceleration

f(R) gravity may lead to an effective dark energy. Consider the FLRW metric, and a perfect fluid, the generalised Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa}{3}\rho_{tot}\,, \quad \left(\frac{\ddot{a}}{a}\right) = -\frac{\kappa}{3}\left(\rho_{tot} + 3p_{tot}\right)$$

$$\rho_{tot} = \rho + \rho_{(c)}\,, \qquad p_{tot} = p + p_{(c)}$$

Where the curvature terms are given by:

$$\rho_{(c)} = \frac{1}{\kappa F(R)}\left\{\frac{1}{2}\left[f(R) - RF(R)\right] - 3\left(\frac{\dot{a}}{a}\right)\dot{R}F'(R)\right\}$$

$$p_{(c)} = \frac{1}{\kappa F(R)}\left\{2\left(\frac{\dot{a}}{a}\right)\dot{R}F'(R) + \ddot{R}F'(R) + \dot{R}^2 F''(R) - \frac{1}{2}\left[f(R) - RF(R)\right]\right\}$$

Late-time acceleration:

$$\rho_{tot} + 3p_{tot} < 0$$!!

3.2 Late-time cosmic acceleration

e.g. vacuum $\rho = p = 0$, effective EOS $\omega_{eff} = p_{(c)} / \rho_{(c)}$

A. Consider $f(R) \propto R^n$, generic power law $a(t) = a_0 (t / t_0)^\alpha$

Results: $\omega_{eff} = -\frac{6n^2 - 7n - 1}{6n^2 - 9n + 3}$, $\alpha = \frac{-2n^2 + 3n - 1}{n - 2}$

Suitable choice of *n* leads to $\omega_{eff} < -1/3$ and late-time acceleration

B. Another example $f(R) = R - \mu^{2(n+1)} / R^n$:

$$\omega_{eff} = -1 + \frac{2(n+2)}{3(2n+1)(n+1)}$$

4. Gauss-Bonnet gravity

- Considering higher-order gravity, motivation consistent with several “quantum gravity” candidates
- String/M-theory predict unusual gravity-matter couplings
- Couple a scalar field with higher order invariants
- String/M-theory predict scalar field couplings with the Gauss-Bonnet invariant important in the appearance of non-singular early time cosmologies
- Apply these motivations to the late-time Universe!

Action of Gauss-Bonnet gravity:

$$S = \int d^4x \sqrt{-g}\left[\frac{R}{2\kappa} - \frac{\lambda}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi) + f(\phi)G\right] + S_m$$

Canonical field: $\lambda = +1$; **phantom field** $\lambda = -1$

Gauss-Bonnet invariant: $G \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$

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FLRW metric, gravitational field equations:

$$\rho_{GB} = \frac{3}{\kappa} H^2, \quad p_{GB} = -\frac{1}{\kappa}\left(3H^2 + 2\dot{H}\right)$$

$$\rho_{GB} = \frac{\lambda}{2}\dot{\phi}^2 + V(\phi) - 24\dot{\phi} f'(\phi) H^3,$$

$$p_{GB} = \frac{\lambda}{2}\dot{\phi}^2 - V(\phi) + 8\frac{\partial}{\partial t}\left(H^2 \dot{f}\right) + 16\dot{\phi} f'(\phi) H^3$$

Equation of motion for the scalar field:

$$\lambda\left(\ddot{\phi}+3H\dot{\phi}\right)+V'(\phi)-24f'(\phi)H^2\left(\dot{H}+H^2\right)=0$$

Define an effective equation of state:

$$\omega_{eff}=\frac{p_{GB}}{\rho_{GB}}=-1-\frac{2\dot{H}}{3H^2}$$

- Exponential scalar potential and scalar-GB coupling:

$$V(\phi)=V_0\, e^{-2\phi/\phi_0},\;\; f(\phi)=f_0\, e^{2\phi/\phi_0}$$

- Scale factor:

$$a(t)=\begin{cases} a_0\, t^{h_0}, & \text{for}\;\; h_0>0 \\ a_0\,(t_s-t)^{h_0}, & \text{for}\;\; h_0<0 \end{cases}$$

- Scalar field:

$$\phi(t)=\begin{cases} \phi_0\, \ln(t/t_1), & \text{for}\;\; h_0>0 \\ \phi_0\, \ln[(t_s-t)/t_1]^{h_0}, & \text{for}\;\; h_0<0 \end{cases}$$

EOS: $\omega_{eff}=-1-\dfrac{2}{3h_0}$ **, if** $h_0<0,\quad \omega_{eff}<-1$ **,** $h_0>0,\quad \omega_{eff}>-1$

5. Some recent work

- $f(R, L_m)$ **theory**: Generalization of all previous *f(R)* gravitational models.
  (Harko, FL, **EPJC 2010**);
  (Harko, FL, **IJMPD 2012**; Honorable Mention in the Gravity Research Foundation Essay Contest 2012)

- Specific application: $f(R,T)$ gravity
  (Harko, FL, Odintsov, Nojiri, **PRD 2011**).

- Generalization: $f(R,T,R_{\mu\nu}T^{\mu\nu})$ gravity
  (Haghani, Harko, FL, Sepangi, Shahidi, **arXiv:1304.5957**)

- **C-theories**: Unification of Einstein and Palatini gravities
  (Amendola, Enqvist, Koivisto, **PRD 2011**).

- Hybrid metric-Palatini theory
  (Harko, Koivisto, FL, Olmo, **PRD 2012**).
  (Capozziello, Harko, FL, Olmo, **arXiv:1305.3756**; Honorable Mention in the Gravity Research Foundation Essay Contest 2013).

5.1 $f(R,T)$ gravity (Harko, FL, Odintsov, Nojiri, PRD 2011)

- **The action is given by:**

$$\boxed{S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa} f(R,T) \; + \; L_m(g^{\mu\nu},\psi)\right]}$$

f (*R,T*) is an arbitrary function of the Ricci scalar, R,
T, trace of the energy-momentum tensor

- Note that the dependence from *T* may be induced by exotic imperfect fluids or quantum effects (conformal anomaly).

- May be considered a relativistically covariant model of interacting dark energy.

5.1 $f(R,T)$ gravity (Harko, FL, Odintsov, Nojiri, PRD 2011)

- Possibility of reconstruction of FRW cosmologies by an appropriate choice of a function $f(T)$ was demonstrated;
- Since the covariant divergence of the stress-energy tensor is non-zero, the motion of massive test particles is non-geodesic;
- Consequently, an extra acceleration, due to the coupling between matter and geometry, is always present;
- The Newtonian limit of the model was investigated, and the expression of the extra-acceleration was also obtained;
- The precession of the perihelion of the planet Mercury was used to obtain a general constraint on the magnitude of the extra-acceleration.

5.2 Hybrid metric-Palatini gravity
(Harko, Koivisto, FL, Olmo, PRD 2012)

- The action is given by:

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,[R + f(\mathcal{R})] + S_m \ ,$$

- Has the scalar-tensor representation:

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[(1+\phi)R + \frac{3}{2\phi}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right] + S_m \ .$$

  - Cosmological applications :
    Capozziello, Harko, Koivisto, FL, Olmo, (JCAP 2013)
  - Dark matter problem in hybrid metric-Palatini gravity:
    Capozziello, Harko, Koivisto, FL, Olmo, (JCAP 2013)
  - Wormhole geometries:
    Capozziello, Harko, Koivisto, FL, Olmo (PRD, 2012)

5.2 Hybrid metric-Palatini gravity
(Harko, Koivisto, FL, Olmo, PRD 2012)

- **Interesting features:**
  - **Predicts the existence of a long-range scalar field, that explains the late-time cosmic acceleration;**
  - **Passes the local tests, even in the presence of a light scalar field.**
  - **Provides an effective geometric alternative to the dark matter paradigm.**

- In a monistic view of Physics, one would expect Nature to somehow choose between the two distinct possibilities offered by the metric and Palatini formalisms.

- We have shown, however, that a theory consistent with observations and combining elements of these two standards is possible.

- Hence gravity admits a diffuse formulation where mixed features of both formalisms allow to successfully address large classes of phenomena.

Conclusions

- Observations imply a late-time cosmic acceleration. But, theory cannot satisfactorily explain it;
- Generalizations of the Einstein-Hilbert (EH) action not such a straightforward procedure:

  - Two distinct classes, metric variational principle and Palatini approach (Both approaches lead to GR for the EH action);

  - Metric-affine theories of gravity: independent connection coupled to matter. Several aspects still completely obscure, such as: exact solutions, post-Newtonian expansions and Solar System tests, cosmological phenomenology, structure structure, application to particle physics, etc.

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  - Metric-affine theories of gravity: independent connection coupled to matter. Several aspects still completely obscure, such as: exact solutions, post-Newtonian expansions and Solar System tests, cosmological phenomenology, structure structure, application to particle physics, etc.
- Confrontation with cosmological, astrophysical and Solar System constraints clarified the difficulty of constructing simple viable models in modified gravity:
  - Viable models need to account for all the cosmological epochs;
  - Solar system tests: problematic issue of "chameleon" mechanism!
- Even if the theory is tailored to fit cosmological observations and pass local tests, problems related with stability arise;
- Theorists need to keep exploring: better models, better observational tests