ENAA 23
Faculdade de Ciências, UL
2013

Compact objects: From white dwarfs to regular black holes

José P. S. Lemos

Centro Multidisciplinar de Astrofisica - CENTRA
Instituto Superior Técnico
Technical University of Lisbon

Lisbon, 18 July 2013

Outline

1. Introduction

- Compact objects: white dwarfs, neutron stars, and black holes (Chandraeskhar 1931; Tolman 1939, Oppenheimer and Volkoff 1939; Oppenheimer and Snyder 1939). In the end of the 1930s the foundations of compact objects were layed out.

- Black holes harbor singularities: undesirable on many grounds (Wheeler 1950s, Penrose 1960s, Hawking 1970s).

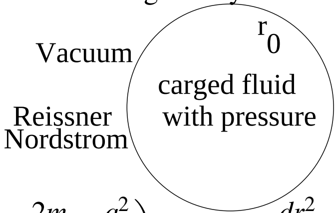
- The idea is to try to find regular black hole solutions, from Einstein's equation ($G = 1\,, c = 1$) $G_{\mu\nu} = 8\pi T_{\mu\nu}$. What to put in $T_{\mu\nu}$?

- Bardeen (1968) showed that from some kind of magnetic matter there was a regular BH solution. Near the center the solution tended to a de Sitter core solution. All subsequent regular BHS are based on Bardeen's proposal, e.g., Lemos, Zanchin (PRD 2011).

- Here we show that Guilfoyle (GRG 1999) electric charged solutions contain compact stars, singular BHs, regular BHs, and quasiblack holes, see Lemos, Zanchin (TBP 2013).

2. Weyl-Guilfoyle ansatz and Guilfoyle's solutions

The compact object: For $r < r_0$, a cold charged pressure fluid is bounded by a spherical surface of radius $r = r_0$. In the electrovacuum region, for $r > r_0$, the metric and the electric potentials are given by extremal Reissner-Nordström solution.



For the outside:

$$ds^2 = -\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right) dt^2 + \frac{dr^2}{1 - \frac{2m}{r} + \frac{q^2}{r^2}} + r^2\, d\Omega\,.$$

$$\phi(r) = \frac{q}{r} + \phi_0\,.$$

For the inside:

$$ds^2 = -B(r)\, dt^2 + A(r)\, dr^2 + r^2\, d\Omega\,.$$

$$\mathscr{A}_\mu = -\phi(r)\, \delta_\mu^0\,, \quad U_\mu = -\sqrt{B(r)}\, \delta_\mu^0\,.$$

Try:

$$B(r) = a\left[-\varepsilon\, \phi(r) + b\right]^2 + c\,, \quad A(r) = \left(1 - \frac{r^2}{R^2}\right)^{-1}$$

$$8\pi \rho_{\rm m}(r) + \frac{Q^2(r)}{r^4} = \frac{3}{R^2}\,.$$

2. Weyl-Guilfoyle ansatz and Guilfoyle's solutions

Can show fluid quantities obey (Lemos, Zanchin PRD 2009):

$$a\,\rho_{\rm e}\,(-\varepsilon\phi + b) = \varepsilon\,(\rho_{\rm m} + 3\,p + \varepsilon\,(1-a)\,\rho_{\rm em})\,B\,, \quad \text{with} \quad \rho_{\rm em} \equiv \frac{1}{8\pi}\frac{(\nabla_i\phi)^2}{B}\,.$$

Such matter systems we called Weyl-Guilfoyle fluids.

Further: put $c=0$ to simplify. Then $\sqrt{a}\,\rho_{\rm e} = \varepsilon\,[\rho_{\rm m} + 3p + (1-a)\,\rho_{\rm em}]\,.$

Joining at r_0:

$$\frac{m}{q} = (1-a)\,\frac{q}{r_0} + \sqrt{a\left(1-\frac{q^2}{r_0^2}\right) + a^2\frac{q^2}{r_0^2}}\,,$$

$$\frac{1}{R^2} = \frac{1}{r_0^3}\left(2m - \frac{q^2}{r_0}\right)\,.$$

$$B(r) = \left[\frac{2-a}{a^{1+1/a}}\,F(r)\right]^{2a/(a-2)}\,,$$

$$8\pi\rho_{\rm m}(r) = \frac{3}{R^2} - \frac{a}{(2-a)^2}\frac{k_0^2\,r^2}{F^2(r)}\,, \quad Q(r) = \frac{\varepsilon\sqrt{a}}{2-a}\frac{k_0\,r^3}{F(r)}\,,$$

$$8\pi p(r) = -\frac{1}{R^2} + \frac{a}{(2-a)^2}\frac{k_0^2\,r^2}{F^2(r)} + \frac{2k_0\,a}{2-a}\frac{\sqrt{1-r^2/R^2}}{F(r)}\,,$$

2. Weyl-Guilfoyle ansatz and Guilfoyle's solutions

where $F(r)$ and $Q(r)$ are defined respectively by

$$F(r) = k_0 R^2 \sqrt{1 - \frac{r^2}{R^2}} - k_1 \,,$$

$$Q(r) = 4\pi \int_0^r \rho_{\rm e}(r) \frac{r^2 dr}{\sqrt{1 - \frac{r^2}{R^2}}} = \frac{r^2}{\sqrt{B(r)}} \sqrt{1 - \frac{r^2}{R^2}} \frac{d\phi(r)}{dr} \,,$$

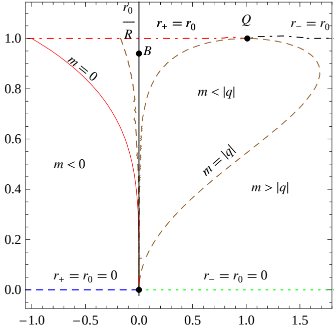
with k_0 and k_1 being integration constants given by

$$k_0 = \frac{|q|\, a^{2/a}}{r_0^3} \left(\frac{m}{q} - \frac{q}{r_0} \right)^{1-2/a} \,,$$

$$k_1 = \sqrt{1 - \frac{r_0^2}{R^2}} \left[k_0 R^2 - \frac{a^{1+1/a}}{2-a} \left(1 - \frac{r_0^2}{R^2} \right)^{-1/a} \right] .$$

3. The plethora of solutions diplayed: stars, regular black holes, quasiblack holes

The plethora:



Plot of $\frac{r_0}{R}$ as a function of $\frac{q^2}{R^2}$.

·Line $m = |q|$ being the same as the line $a = 1$, represents Bonnor stars and Majumdar-Papapetrou matter.

·Inside: overcharged tension stars. Outside: charged stars and BHs.

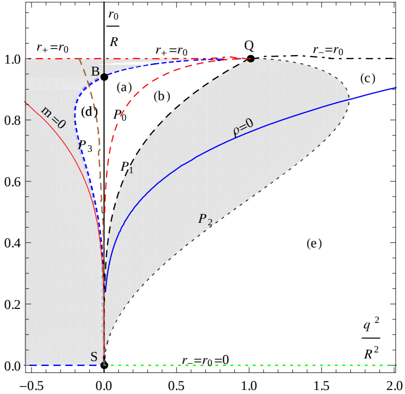
·B: Buchdahl. S: Schwarzschild. Q: quasiblack hole (Lemos, Zaslavskii 2007-2013).

·$r_+ = r_0$ weird BHs. $r_- = r_0$ regular BHs, r_0 at the Cauchy horizon.

·Allow $\frac{q^2}{R^2}$, imaginary charges. One horizon. No BHs up to top line. Line $r_0 = 0$ a naked singularity. (See Einstein, Rosen 1935 for $m = 0$, $q^2 < 0$ bridge.)

3. The plethora of solutions diplayed: stars, regular black holes, quasiblack holes

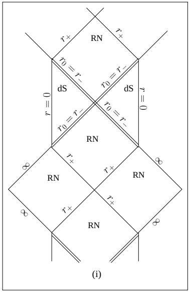
With singular detail:

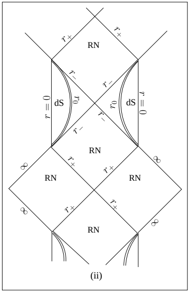


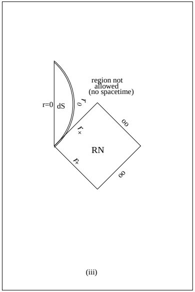
Regions (a) and (d): Charged stars. Region (b): Overcharged tension stars. Regions (c) and (e): Regular BHs with timelike boundary. Line $r_- = r_0$: Extremal regular BHs. Point Q: The quasiblack hole with pressure. Line $r_+ = r_0$: Singular (weird) BHs. Lines P_0 and P_2: Charged dust stars. Line P_1: Singular at center. Line $\rho = 0$: $\rho = 0$. Line BS: Neutral stars. Point B: The Buchdahl limit. Point S: Schwarzschild BHs. Line $r_0 = 0$: Special cases.

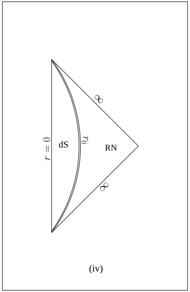
3. *The plethora of solutions diplayed: stars, regular black holes, quasiblack holes*

Carter-Penrose diagrams:









4. Conclusions

· Compacts stars are always welcome.

· Quasiblack holes display outstanding properties wich enable to calculate the BH entropy by thermodynamic means (Lemos, Zaslavski PRD 2011, PLB 2012).

· Regular black holes help in the understanding of the black hole interior and the singularity problem.