Coupled 3-form Dark Energy

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Koivisto & Nunes, arXiv:1212.2541

What is a three-form?

Totally antisymmetric tensor with three indeces

$$A_{ijk} = -A_{jik}$$

For example, a three-form defines the cross product

$$(\vec{a} \times \vec{b})_i = \epsilon_{ijk}\, a_j b_k$$

where ϵ_{ijk} is the Levi-Civita symbol.

Why forms?

1. To test the robustness of scalars.
   Is it the only natural possibility?

2. Vectors and two-forms,
   Support anisotropy;
   Require non-minimal couplings $\longrightarrow$ possible instability.

   Three-forms,
   Viable isotropic inflation.

3. They exist in fundamental theories
   String theory.

Three-form action

Action for the three-form $A_{\mu\nu\rho}$

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} - \frac{1}{48}F^2(A) - V(A^2) - \sum_a m_a(A^2)\delta(x - x(\lambda))\sqrt{\frac{\dot{x}^2}{-g}}\right)$$

where

$$F_{\mu\nu\rho\sigma} = 4\nabla_{[\mu}A_{\nu\rho\sigma]} = \nabla_\mu A_{\nu\rho\sigma} - \nabla_\sigma A_{\mu\nu\rho} + \nabla_\rho A_{\sigma\mu\nu} - \nabla_\nu A_{\rho\sigma\mu}$$

We have the equations of motion:

$$\nabla \cdot F = 12\left(V'(A^2) + 2\rho\frac{m'(A^2)}{m(A^2)}\right)A$$

and due to antisymmetry we have the additional constraints:

$$\nabla \cdot \left(V'(A^2) + 2\rho\frac{m'(A^2)}{m(A^2)}\right)A = 0$$

Equations of motion

Consider flat FRW cosmology:

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

Most general three-form compatible with FRW:

$$A_{ijk} = a^3(t)\epsilon_{ijk}\chi(t)$$

Equations of motion of the field χ with $f \equiv 2m_{,\chi}/m$

$$\ddot{\chi} + 3H\dot{\chi} + V_{,\chi} + 3\dot{H}\chi = -\kappa\rho_m f$$

Equation of motion of dark matter fluid:

$$\dot{\rho}_m + 3H\rho_m = \kappa\rho_m f$$

Equations of motion

Friedmann equation

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{3}\left(\frac{1}{2}(\dot{\chi} + 3H\chi)^2 + V(\chi) + \rho_m\right)$$

can also write:

$$H^2 = \frac{\kappa^2}{3}\frac{V + \rho_m}{1 - \kappa^2(\chi' + 3\chi)^2/6}$$

$\Rightarrow \qquad \kappa|\chi' + 3\chi| < \sqrt{6}. \qquad\qquad {}' = d/d\ln a.$

Evolution of the Hubble rate:

$$\dot{H} = -\frac{\kappa^2}{2}\left(V_{,\chi}\chi + (1 + \kappa f\chi)\rho_m\right)$$

Equation of state parameter of χ:

$$w_\chi = -1 + \frac{V_{,\chi} + \kappa f\rho_m}{\rho_\chi}\chi$$

Effective potential

$$\ddot{\chi} + 3H\dot{\chi} + V_{,\chi} + 3\dot{H}\chi = -\kappa\rho_m f$$

$$V_{\text{eff},\chi} = V_{,\chi}\left(1 - \frac{3}{2}(\kappa\chi)^2\right) - \frac{3}{2}\kappa^2\rho_m\chi + \kappa f\rho_m\left(1 - \frac{3}{2}(\kappa\chi)^2\right)$$

We are going to study 4 cases:

(i) $V = 0$, $f = 0$;

(ii) $V = 0$, $f \neq 0$;

(iii) $V \neq 0$, $f = 0$;

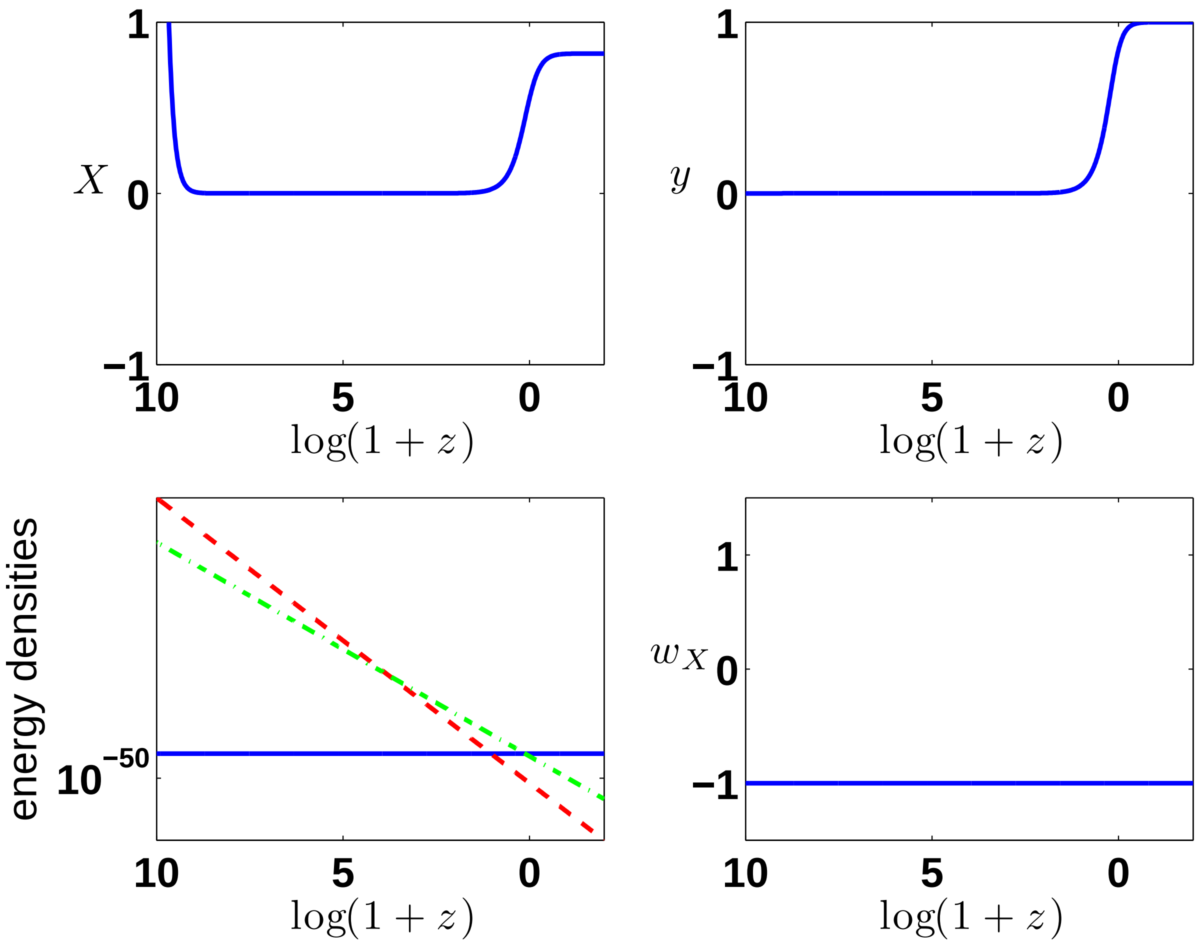
(iv) $V \neq 0$, $f \neq 0$.

Case: $V = 0$, $f = 0$;

$$y_i \equiv \chi_i' + 3\chi_i \neq 0 \qquad \text{otherwise} \qquad \rho_\chi \equiv 0.$$

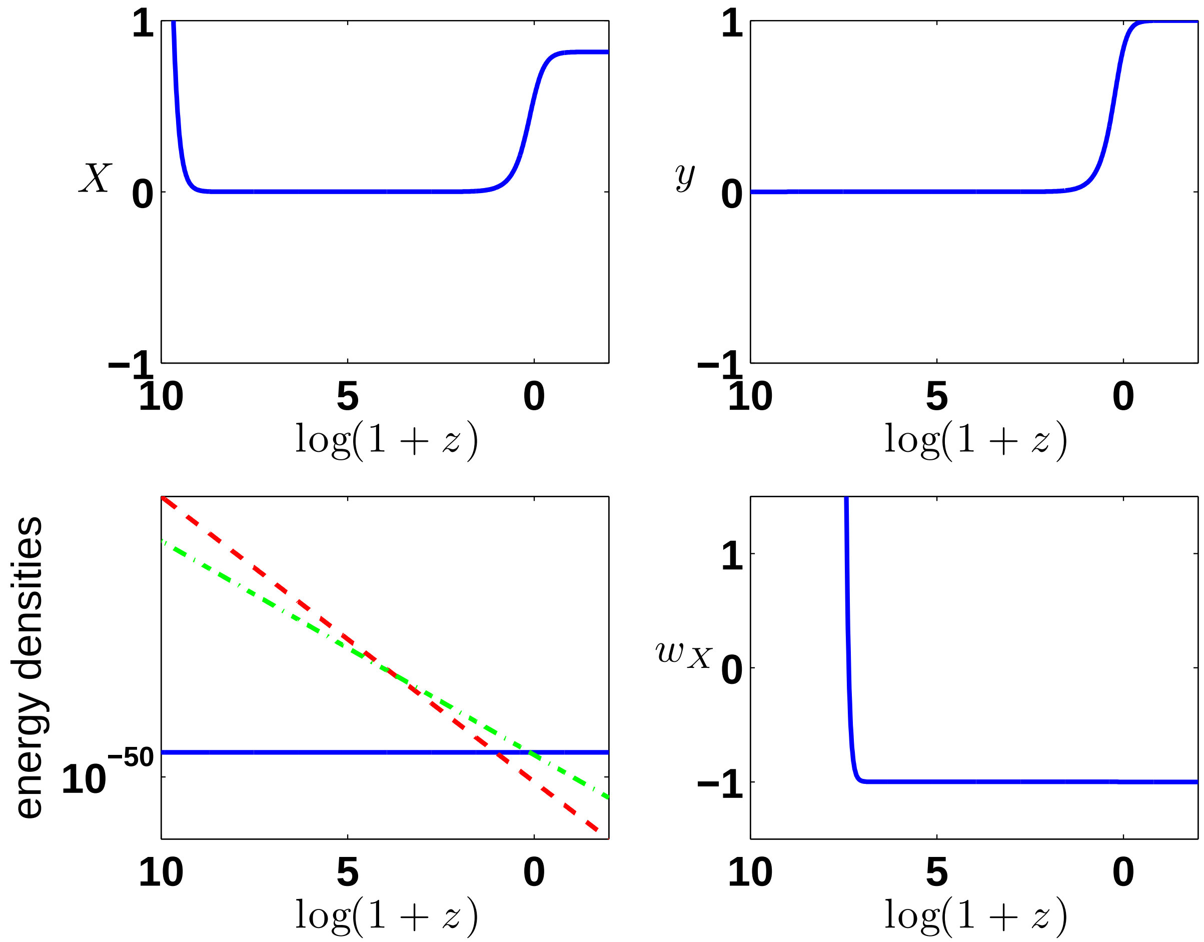
$$w_\chi \equiv -1$$

It is a cosmological constant!



Case: $V = 0$, $f \neq 0$;

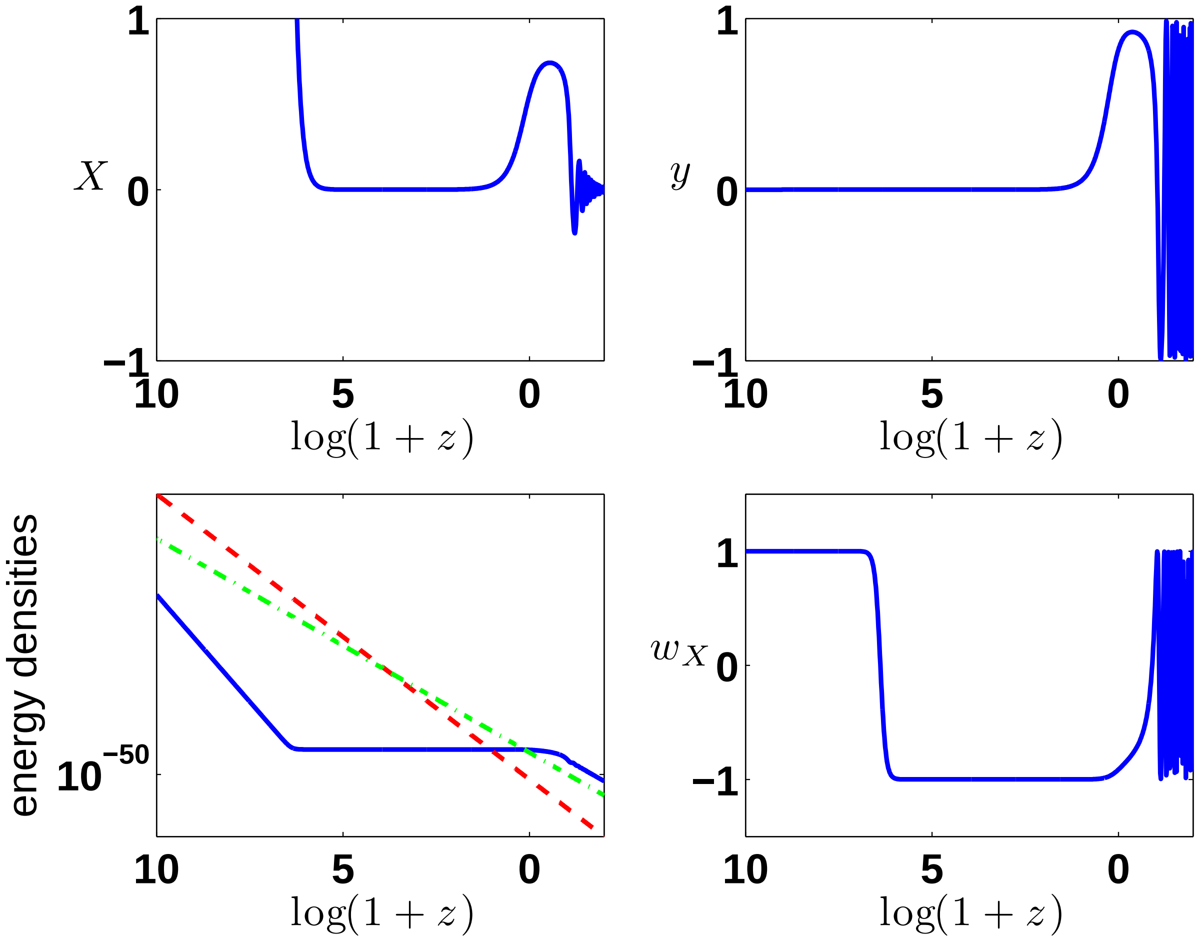
$$w_\chi = -1 + \frac{\kappa f \rho_m}{\rho_\chi}$$



Case: $V \neq 0$, $f = 0$;

With $V = V_0\chi^2$;

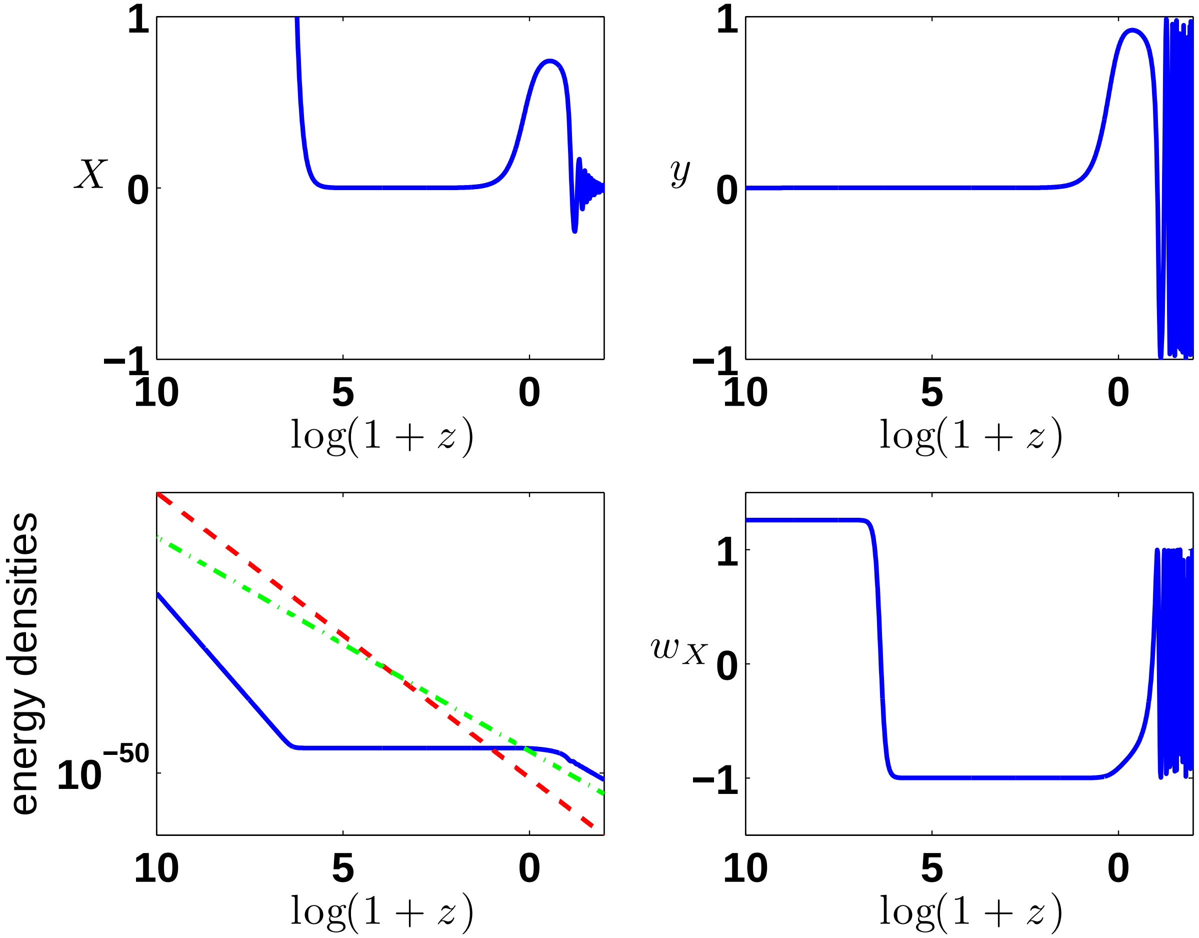
$$w_\chi = -1 + \frac{V_{,\chi}}{\rho_\chi}\chi$$



Case: $V \neq 0$, $f \neq 0$;

With $V = V_0\chi^2$;

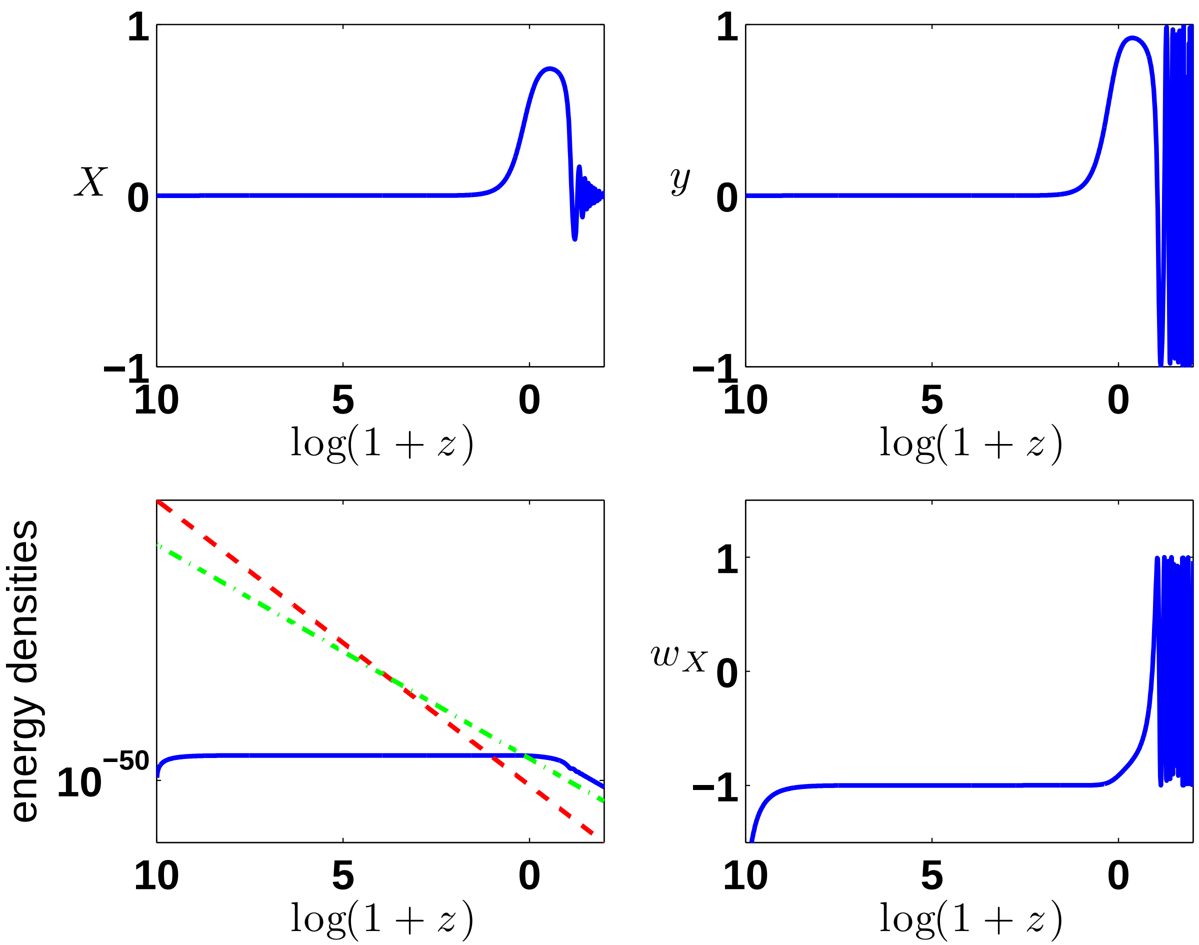
$$w_\chi = -1 + \frac{V_{,\chi} + \kappa f \rho_m}{\rho_\chi}\chi$$



Case: $V \neq 0$, $f \neq 0$;

With $V = V_0\chi^2$ and $\chi_i = \chi_i' = 0$

$$w_\chi = -1 + \frac{V_{,\chi} + \kappa f \rho_m}{\rho_\chi}\chi$$



Newtonian limit of linear perturbations

Linear evolution of matter density perturbations

$$\ddot{\delta}_m + \left(2H + \kappa f \dot{\chi} - \frac{2\dot{F}}{1-F}\right)\dot{\delta}_m = \left(\frac{\kappa_{\rm eff}^2}{2}\rho_m - \frac{k^2}{a^2}c_{\rm eff}^2\right)\delta_m$$

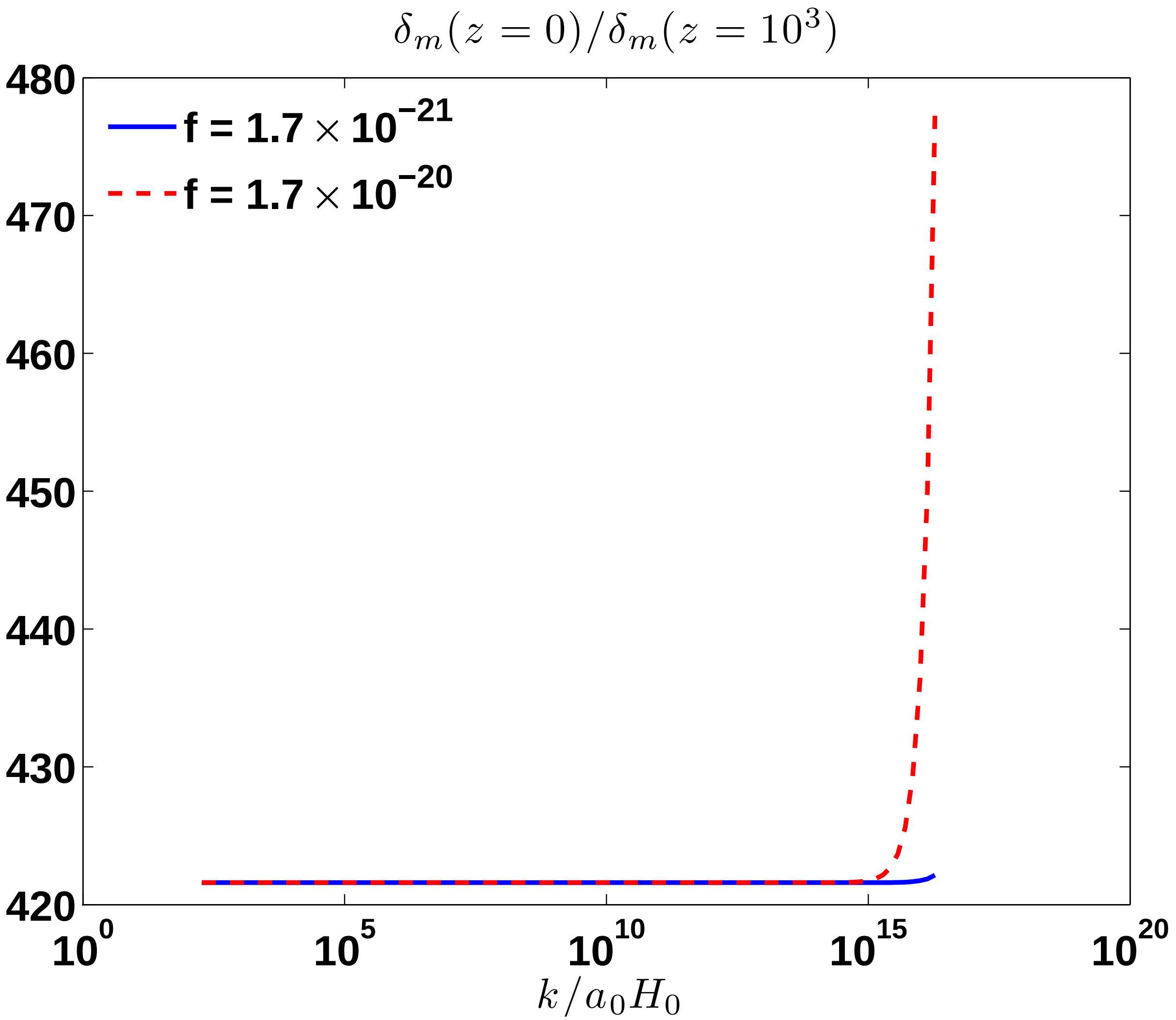
$$\kappa_{\rm eff}^2 = \frac{\kappa^2}{1-F}\left[1 + \frac{2}{\kappa^2\rho_m}\left(\ddot{F} + (2H + \kappa f\dot{\chi})\dot{F} - \frac{\kappa^2}{2}\frac{V_{,\chi}\kappa f \rho_m}{V_{,\chi\chi} + \kappa f_{,\chi}\rho_m}\right)\right]$$

$$c_{\rm eff}^2 = \frac{F}{1-F}\,, \qquad\qquad F \equiv -\frac{\kappa^2 f^2 \rho_m}{V_{,\chi\chi} + \kappa f_{,\chi}\rho_m}$$

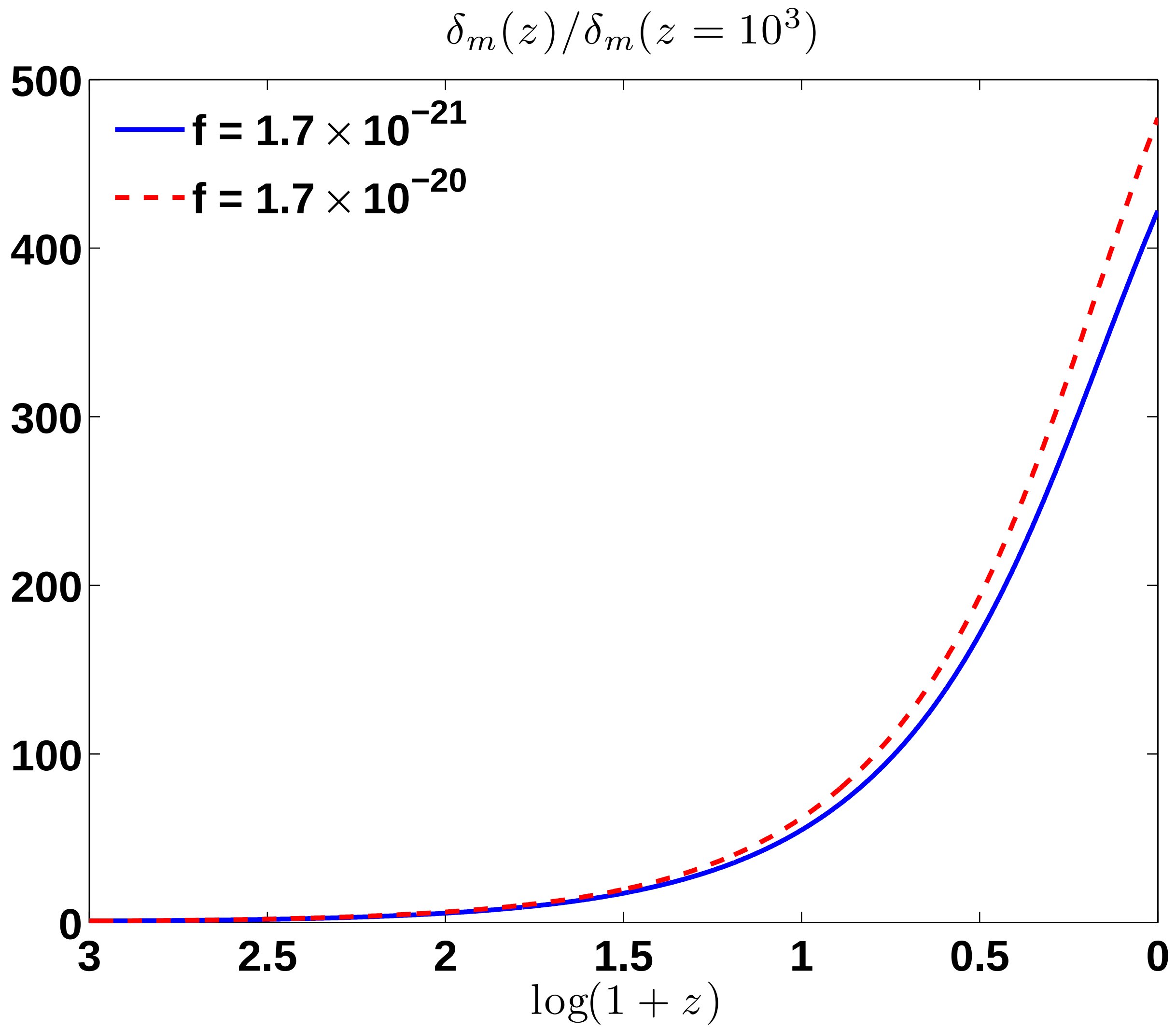
$F < 0 \quad \Rightarrow \quad c_{\rm eff}^2 < 0 \quad \Rightarrow$

for sufficiently large modes there is extra source term for growth of perturbations!

Growth at a given time



Growth for a given mode



Summary

- Three-forms possess accelerating attractors and saddle points which can describe three-form driven inflation or dark energy;

- Three-forms can give rise to viable cosmological scenarios with potentially observable signatures distinct from standard single scalar field models;

- In the presence of a coupling to dark matter, growth of structure is enhanced for small scales.

Further afield

- Formation of structure (N-body simulations, perturbative approach);
- Other couplings to matter;
- Screening mechanisms.