Three-form Cosmology

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Outline

1- Fields and cosmology

2- Three-form theory

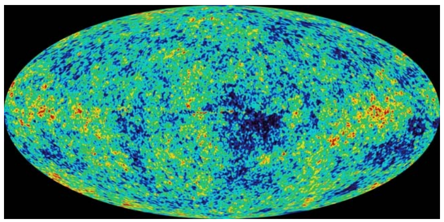
3- Static limit

4- Dynamic limit

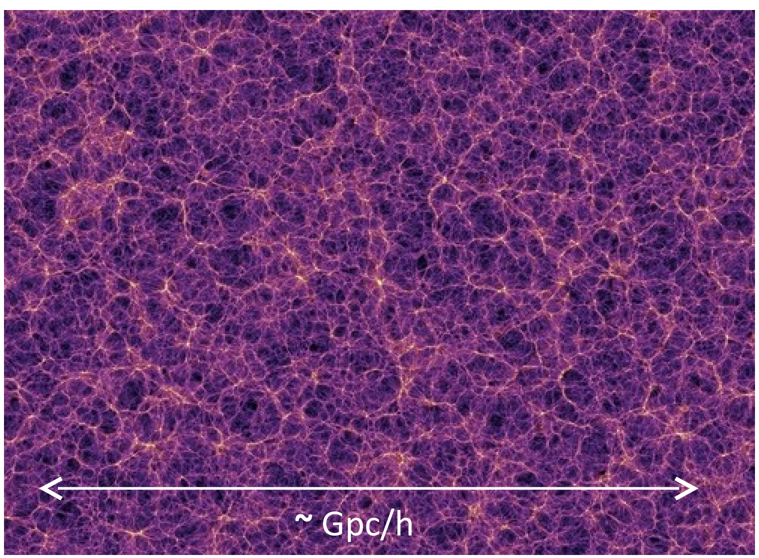
5- Conclusions

Fields and comology

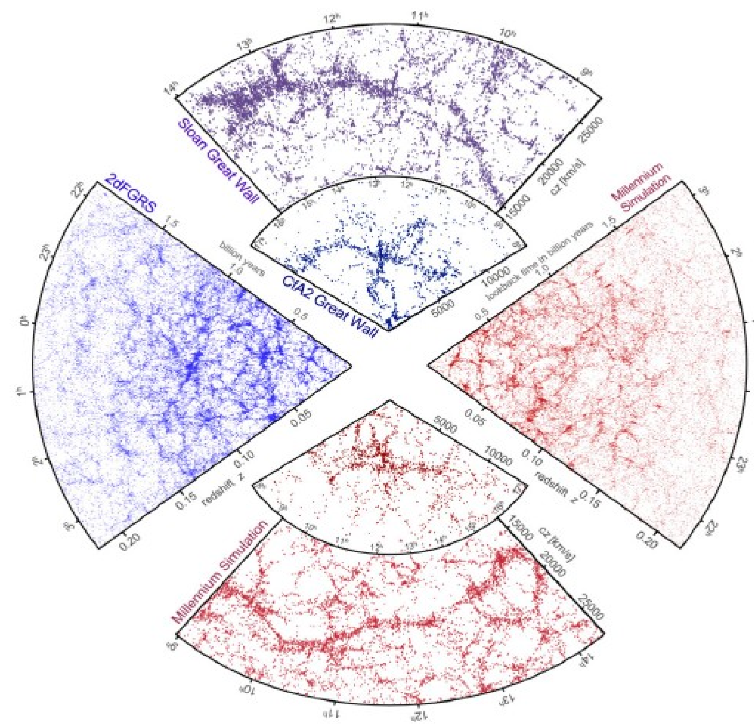
First observation: from CMB and LSS we know that the universe is flat and statistically homogeneous and isotropic with correlation on all scales, even though scales separated by more that one degree had no time to interact after recombination



WMAP



Millennium simulation



LSS from simulations and observations

How do we explain these features?
We postulate a period of very fast expansion in the early universe, driven by a field dubbed as the "inflaton"

$$\Omega_k \doteq \frac{K}{(aH)^2} \sim \dot{a}^{-2} \qquad \frac{d^c(t)}{d^c(t_0)} = \frac{a_0 H_0}{aH} \sim \dot{a}^{-1}$$

The equations of motion of the inflaton are:

$$H^2 = \frac{8\pi G}{3}\rho_\phi = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \qquad \ddot{\phi} + 3H\dot{\phi} = -V'[\phi]$$

A quadratic potentil gives rise to an exponential expansion:

$$V = \frac{1}{2}m^2\phi^2 \qquad a \sim a_i \exp\left[\sqrt{\frac{4\pi G}{3}} m(\phi_i t - \frac{m}{4\sqrt{3\pi G}}t^2)\right]$$

Candiates for inflation: Starobinsky, non-minimally coupled Higgs, three-forms, braneworld, etc (hope this doesn't offend anyone)

Second observation: the universe is expanding and accelerating

How do we know it?
Supernovae, CMB, etc

How do we model it? We take our best theory of gravity up to date, GR, given an ideal fluid matter model and the FRW metric, we solve Einstein equation and we get:

$$\frac{\ddot{a}}{a} = -4\pi G\rho + \frac{\Lambda}{3}$$

$$H^2 = \frac{8\pi G}{3}\rho$$

Alternative dark energy models: scalar fields (quintessence, dilaton, etc), non-scalar fields (vectors, three-forms, etc), modified gravity, inhomogeneous models (large void, backreaction)

2- Three-form theory

The action of a three-form theory conformally coupled to matter is:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{48}F^2 - V(A^2)\right] + \int d^4x\sqrt{-\hat{g}}\hat{\mathcal{L}}_m$$

Where:

$$F_{\alpha\beta\gamma\delta} = 16\nabla_{[\alpha}A_{\beta\gamma\delta]} \qquad \hat{g}_{\mu\nu} = \Omega^2\left(A^2, (\nabla A)^2\right)g_{\mu\nu}$$

From this we derive the equations of motion:

$$\nabla_\mu F^{\mu\alpha\beta\gamma} = 12\left(V'(A^2) + \rho\Omega'(A^2)\right)A^{\alpha\beta\gamma} - \nabla_\epsilon(\rho\nabla^\epsilon A^{\alpha\beta\gamma}\Omega'((\nabla A)^2))$$

Where the three-form admits the general parametrization:

$$A_{0ij} = a(t)\epsilon_{ijk}i\zeta^k(t,x,y,z) \qquad A_{ijk} = a^3(t)\epsilon_{ijk}\chi(t,x,y,z)$$

3- Static limit

In this limit the metric is of the Minkovski type and the three-form is parametrized as:

$$A_{0ij} = \epsilon_{ijk} i\phi^k(x,y,z) \qquad A_{ijk} = \epsilon_{ijk}\xi(x,y,z)$$

The equations for the spatial fields are:

$$\xi\left(V'[\omega_1] + \rho\Omega'[\omega_1]\right) = 12\rho\nabla^2\xi\Omega'[\omega_2] + 144\rho\xi'\Omega''[\omega_2]\left(\xi'\nabla^2\xi - \phi'\nabla^2\phi\right)$$

$$\nabla^2\phi = 12\phi\left(V'[\omega_1] + \rho\Omega'[\omega_1]\right) + 144\rho\phi'\Omega''[\omega_2]\left(\phi'\nabla^2\phi - \xi'\xi''\right)$$

Where:

$$\omega_1 \equiv A^2 = 6\left(\phi^2 + \xi^2\right) \quad \omega_2 \equiv (\nabla A)^2 = 6\left(\phi'^2 + \xi'^2\right)$$

They reduce to:

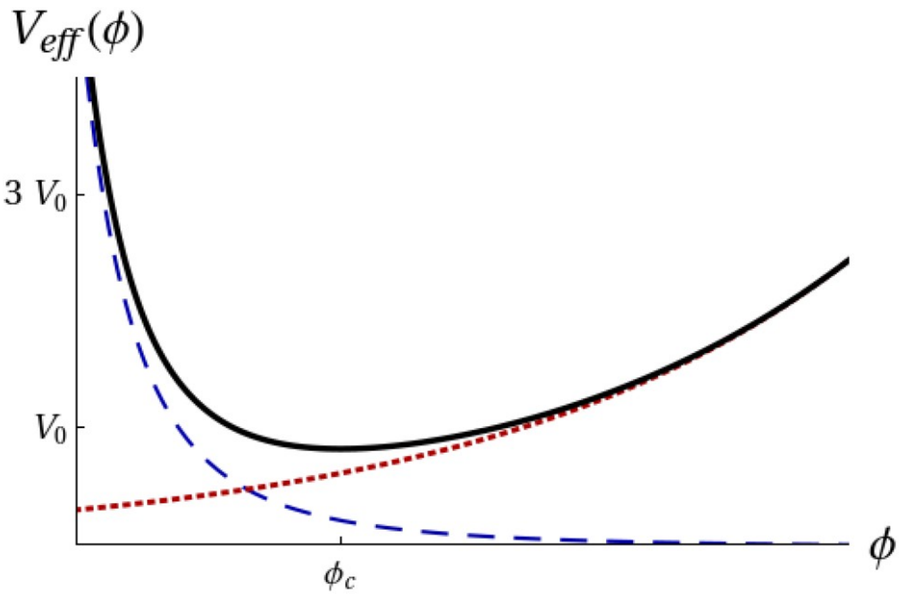
$$\nabla^2\phi = \frac{V'[\phi] + \rho\Omega'[\phi]}{1 + 144\rho\phi'^2\Omega''[6\phi'^2]}$$

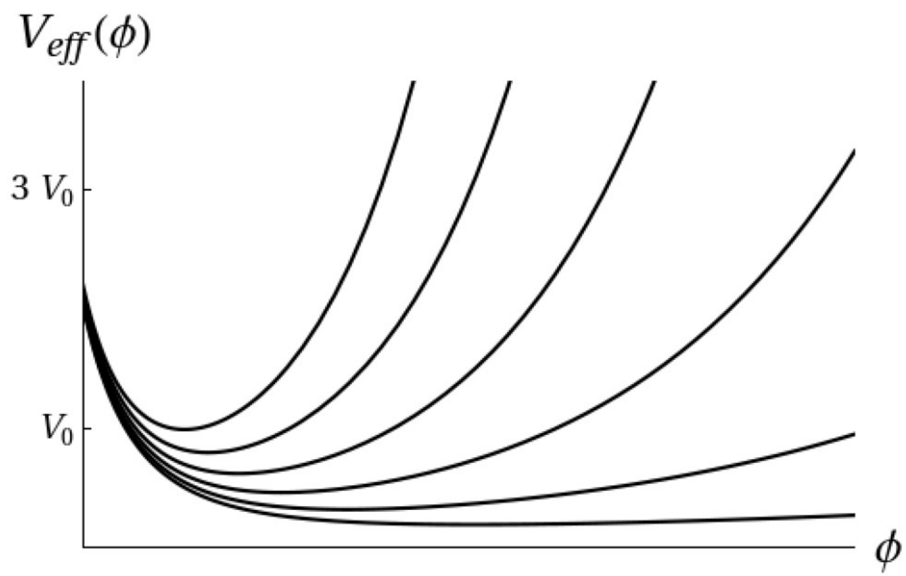
Which simplifies, in the non-derivative case, to the well knwon scalar chameleon equation:

$$\nabla^2\phi = V'[\phi] + \tilde{\rho}\Omega'[\phi]$$

The mass of the chameleon field is:

$$m_\phi^2 \doteq V''_{eff}[\phi_{min}] = V''[\phi_{min}] + \rho\Omega''[\phi_{min}]$$



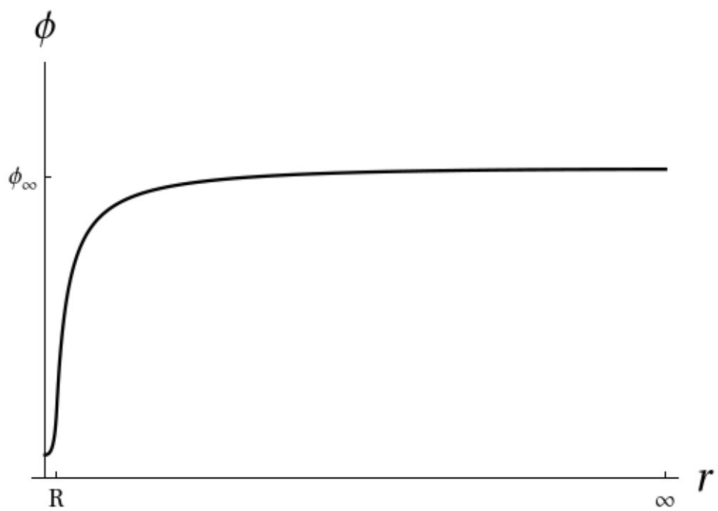


A solution to the chameleon equation, around the earth, is:

$$\phi(r) \sim \left(\frac{\Delta R}{R}\right) \frac{M_c e^{-m_\infty (r-R)}}{r}$$

The fifth-force is suppressed compared to the Newtonian force

The field profile is:



4- Dynamic limit

In this limit the three-form is parametrized as:

$$A_{0ij} = a(t)\epsilon_{ijk} i\zeta^k(t) \qquad A_{ijk} = a^3(t)\epsilon_{ijk}\chi(t)$$

The equations of motion are somewhat complicated, however they reduce to:

$$\ddot{\chi} + 3\left(\chi H\right)^{\cdot} = -\left(V'[\chi] + \rho\Omega'[\chi]\right)$$

While the Friedmann equations are:

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\left(\dot{\chi} + 3H\chi\right) + V(\chi) + \rho\right]$$

$$\frac{\ddot{a}}{a} + \frac{H}{2} = 4\pi G\left[\frac{1}{2}\left(\dot{\chi} + 3H\chi\right) + V(\chi) - \chi(V'[\chi] + \rho\Omega'[\chi]\right]$$

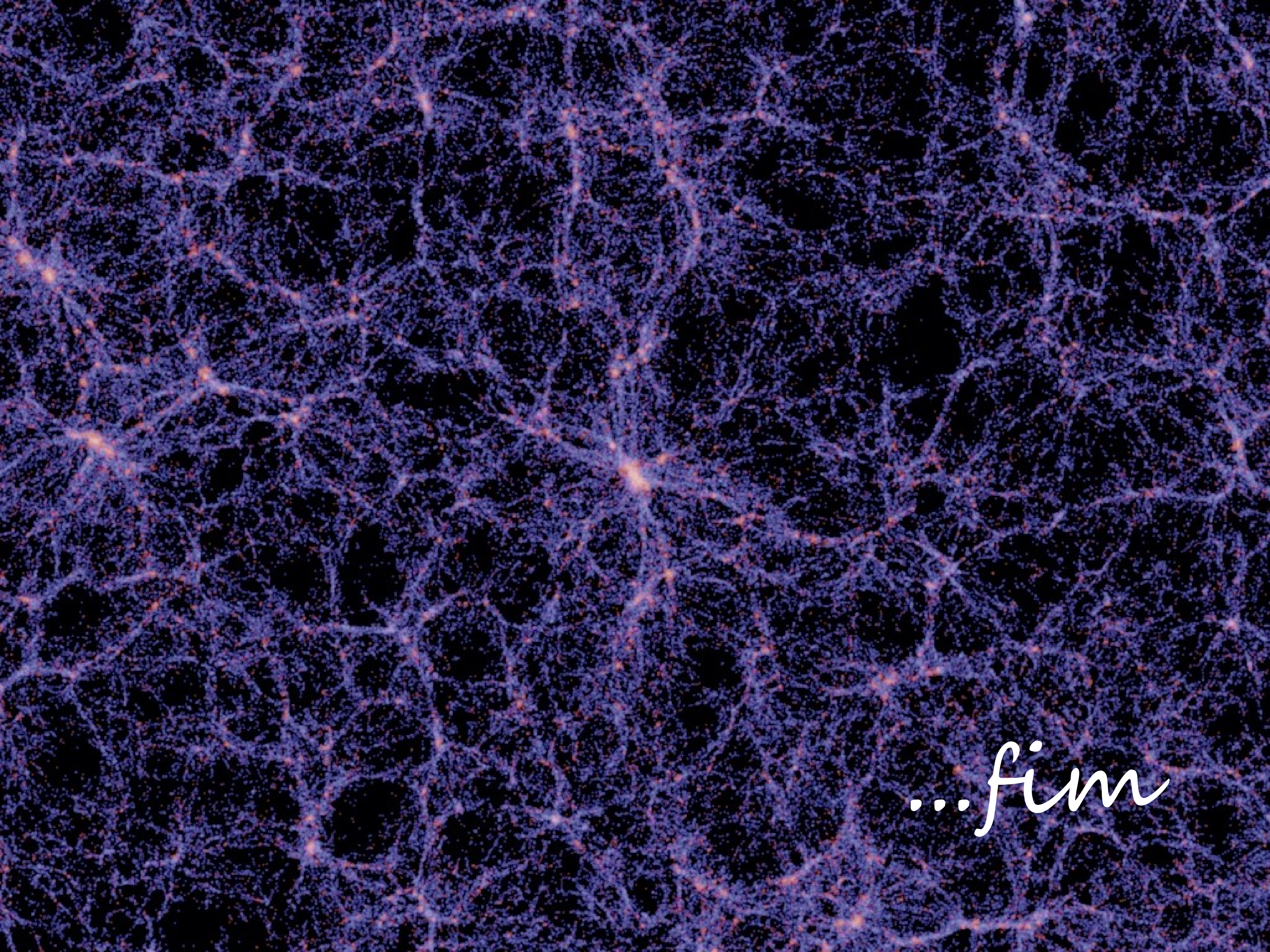
In the case of no coupling, we can study three-form inflationary models. We can on the same theoretical ground study dark every from three-forms (this will be discussed in the next lecture)

5- Conclusion

There are many competing models which aim to expalin inflation and dark energy, one of this relies on three-forms

From the particle physics point of view, the fifth-force coming from three-forms can be screened on solar system scales by the so-called chameleon mechanism

For the future it would be interesting to study three-forms with derivative couplings, and to track the behaviour of a three-form field from inflation up to late times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